This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

STEVENS AND HOLE'S GRADE LESSON-BOOKS.

Now in course of publication, in 12mo.

THE

GRADE LESSON-BOOKS,

IN SIX STANDARDS;

EACH EMBRACING

Reading, Spelling, Writing, Arithmetic, and Exercises for Dictation.

ESPECIALLY ADAPTED TO MEET THE REQUIREMENTS OF THE 'REVISED CODE.

BY E. T. STEVENS,
Associate of King's College, London;
AND
CHARLES HOLE,
Head Master, Loughborough Collegiate School, Brixton; late Master of St. Thomas's Collegiate School, Colombo, Ceylon.

The FIRST STANDARD, pp. 96, price 9*d.* cloth.
The SECOND STANDARD, pp. 128, price 1*s.* cloth.
The THIRD STANDARD, pp. 160, price 1*s.* cloth.
The FOURTH STANDARD, pp. 160, price 1*s.* cloth.

⁂ The FIFTH and SIXTH STANDARDS, completing the course, are nearly ready.

'THESE little books are especially adapted to meet the requirements of the Revised Code. It is scarcely possible to produce a better method of teaching little children how to read and write and do sums in arithmetic. Gentlemen's children have seldom been taught these first essential elements of learning half so well as the children of the poor may now be taught with the aid of these very cleverly composed primers.' GLOBE.

'THIS series of school-books is professedly adapted to the new Minutes of Council on Education. Each part, or standard, as it is termed, contains all the matter which is required in order to qualify for examination at each stage of progress, and specimens are given of the manner. This must be a great relief to the pupil and of much assistance to the teacher. The first part begins very simply, and advances by clearly marked gradations.... In this respect, the Grade Lesson-books are all we can desire. They contain such narratives as a child at school would delight in. The first three of the series are already published; we shall most gladly welcome the remaining three. We would only further add that their cheapness as well as their neat and portable form are recommendations not to be overlooked.' CALEDONIAN MERCURY.

By the same Authors, preparing for publication, in 12mo. with numerous attractive Woodcuts,

The GRADE LESSON-BOOK PRIMER, for the use of Infant Schools; introductory to the *Grade Lesson-Books*, now in course of publication as above.

THIS *Primer* is [illegible] *de Lesson-Books.* The child- introduction to t[illegible] at book are led, by the easiest same systematic [illegible] from one difficulty to another; *monosyllables* be[illegible] of words being, however, con- *which characteris*[illegible] easier and more familiar ones.

London: [illegible] . Paternoster Row.

THE CIVIL SERVICE ARITHMETIC,

CONTAINING ONE THOUSAND THREE HUNDRED
QUESTIONS PROPOSED BY THE CIVIL SERVICE COMMISSIONERS, WITH
SOLUTIONS OF SOME OF THE MOST DIFFICULT.

FOR THE USE OF SCHOOLS AND COLLEGES.

BY

ROBERT JOHNSTON,

AUTHOR OF
'THE SCHOOL ARITHMETIC' ETC.

LONDON:
LONGMAN, GREEN, LONGMAN, ROBERTS, & GREEN.
1863.

~~180. c. 171~~
181. g. 135.

LONDON
PRINTED BY SPOTTISWOODE AND CO.
NEW-STREET SQUARE

BIBLIOTHECA · BODLEIANA ·

BY PERMISSION.

◆

TO

SIR ROBERT PEEL, BART.

CHIEF SECRETARY FOR IRELAND,

&c. &c.

THE MUNIFICENT PATRON

AND ARDENT ADVOCATE OF EDUCATION,

THIS TREATISE

IS

MOST RESPECTFULLY DEDICATED

BY

THE AUTHOR.

PREFACE.

THE system of Competitive Examination introduced into the Civil Service has rendered a modification in the character of most of our elementary school-books absolutely necessary. To effect this object in the Arithmetical Department this treatise has been compiled. It contains almost all the questions on Arithmetic given in the Civil Service Reports, and several others which were obtained from parties who underwent the examinations at which they were proposed. The questions are arranged under the various rules, and referred as far as possible to the Department of the Civil Service in which they were proposed. Some of the most difficult are worked as examples, and every pains has been taken to render the treatise suitable for the requirements of the Public Service.

⁂ Should the reader find any errors in this work, he will greatly oblige the Author by communicating with the Publishers.

CONTENTS.

PAGE

Definitions, Notation and Numeration 1
Addition 5
Subtraction 9
Multiplication 10
Division 16
Weights and Measures 24
Miscellaneous Questions 26
Reduction 28
Compound Addition 34
Compound Subtraction 42
Compound Multiplication 46
Compound Division 49
Duodecimals 54
Miscellaneous Exercises 58
VULGAR FRACTIONS 62
    Reduction 64
    Greatest Common Measure 66
    Least Common Multiple 68
    Addition 72
    Subtraction 75
    Multiplication 77
    Division 79
DECIMALS 82
    Addition 84
    Subtraction 85
    Multiplication 86
    Division 88
    Circulating Decimals 94

PAGE
Miscellaneous Exercises 99
Practice 101
Miscellaneous Exercises 111
Simple Proportion 114
Compound Proportion 125
Miscellaneous Exercises 127
The Chain Rule 131
Partnership 133
Simple Interest 134
Discount 142
Commission, &c. 145
Equation of Payments 146
The Stocks 148
Profit and Loss 154
Averages and Percentages 157
Involution 161
Evolution 163
Progressions 167
Compound Interest 169
Logarithms 171
Examination Papers 176
Miscellaneous Questions 185
Examination Papers 202
Questions given at London University &c. 205
Answers 209

ERRATA.

Page 104, last line, *for* 204, *read* 205.
„ 192, second line, *for* 16*s.* 9*d.* *read* £16 9*s.*

THE CIVIL SERVICE

ARITHMETIC.

1. QUANTITY is a term applied to anything susceptible of being increased or diminished.

2. A UNIT is one, or an individual thing of the quantity under consideration.

3. The term NUMBER may be applied to a unit or an assemblage of units of any magnitude whatever.

4. ARITHMETIC is the science of numbers, and is either theoretical or practical. *Theoretical Arithmetic* explains the nature, properties and relations of numbers: *Practical Arithmetic* applies those principles to the various purposes of life.

5. All numbers are expressed by the following ten characters or figures, either *singly* or in *combination*, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, read, *nought* or *cypher*, *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*.

6. When any one of these figures stands by itself, it has what is called its *simple* value, which never varies. When two or more figures are used to express a number, all except that in the *units' place* (to the right of the other figures), have what is termed a *local* value, which *varies*

according to the distance of the figure from the units' place. Thus, if a figure stand *one* place to the left of the units' place, its *local* value is ten times its *simple* value; if *two* places to the left, its *local* value is one hundred times its *simple* value, and so on.

7. 1, 3, 5, 7, &c. are called *odd* numbers: 2, 4, 6, 8, &c. are called *even* numbers.

8. Notation is the art of expressing numbers by characters or figures: Numeration, the art of reading them when written.

9. For the convenience of reading and writing numbers, we divide them into *periods*, each consisting of three places of figures, as in the following Table.

NOTATION AND NUMERATION TABLE.

Millions.			Thousands.			Units.		
0	0	0,	0	0	0,	0	0	0.
...	..	.	...	..	.	...	..	.

This Table has three periods, *Units*, *Thousands*, *Millions*, consisting of three places each, which, commencing at the right in every period, are invariably read, *units*, *tens*, *hundreds*. To show the *sameness* of the places which form each period, the *units' place* is marked by one dot, the *tens'* by two, and the *hundreds'* by three dots. The *periods* are all *alike* as regards the places which form them. It follows, that if we can write numbers consisting of three places of figures, we can write any number, however great. For instance, to write *six*, we place it in the *first place* to the right: because it is 6 *ones or units*, we must place it in the '*units' period*.' Again, if we wish to write *six thousand*, we have merely to remove the six from its position in the units' period to a similar one in the *thousands' period*; thus, 6,000, that is, we pass over the first period, the places of which must be filled up by cyphers; the great use of the cypher being to fill up the places which would otherwise be vacant. In the same manner, if we have to write *three-hundred and six units*, *three-hundred and six thousand*, *and three-hundred and six millions;* we have, *respectively*, 306; 306,000; 306,000,000; each number being placed in its proper period. The next period to the left, after millions, is called *billions*, next *trillions*, and so on, to an *indefinite* extent: the number of periods is *indefinite*; the number of places only *three.*

10. TO READ ANY NUMBER.

RULE.—1°. Commencing at the right, divide it into periods, allowing three places to each.

2°. Then read each period from the left, adding the name of the period to the number which its figures express.

To read 43506403; by dividing it into periods, we have 43,506,403, which we read forty-three, five hundred and six, four hundred and three. Now, by adding the names of the periods, we have forty-three *millions*, five hundred and six *thousand*, four hundred and three *ones* or *units.*

11. Ours being a *decimal* system of Notation, the figures increase to the left in a *ten-fold* ratio: in other words, *ten units* make *one ten*, that is, one unit in the tens' place; ten of these units in the tens' place make one hundred, and so on.

From the nature of this NOTATION, it is evident, that if we remove a figure *one* place to the left, we make its value *ten times* greater. Thus 60 is equal to ten times 6. If we remove it *two* places to the left, we make its value *one hundred times* greater, and so on: thus 600 is equal to one hundred times 6. On the contrary, if we remove a figure *one place* to the right, we decrease its value *ten-fold*; if we remove it *two places* to the right, we decrease its value *one hundred-fold.* In the one case, the *units* become *tens*, the *tens*, *hundreds*, &c.: in the other the *hundreds* become *tens*, the *tens*, *units.*

A NOUGHT placed to the *left* of a figure or number has *no effect,* since it does not alter the position with respect to the *units' place.*

I.

Write in figures the following numbers:[1]

1. Five hundred and ninety.
2. One thousand four hundred and three.
3. Two thousand and seventy-seven.
4. Four thousand five hundred and seventy.
5. Eight thousand and nine.
6. Nine thousand and ninety-seven.

Post Office (Provincial).

7. Four hundred and sixty-one thousand and one.
8. Two millions, five thousand and eighty-seven.
9. Twenty-seven millions, seven hundred and six.

Apprentices in Dockyards.

10. Eight hundred and forty thousand and twenty.
11. Ten millions four thousand and eight.
12. Eleven hundred and two millions seventy thousand and fifty.

Committee of Council on Education.

13. Ninety thousand and forty.
14. Three hundred and four millions, forty thousand and four.

Various Examinations for offices where 'Elementary Arithmetic' is to be known.

15. Six hundred and three thousand and fifty.
16. Seven millions three thousand and forty.
17. Eight thousand millions one thousand and two.
18. Five hundred thousand four hundred and one.
19. Thirty millions seven thousand and forty.
20. Five hundred and six millions and twenty.
21. Four millions five hundred and five.
22. Eight thousand millions six hundred.
23. Ten millions twenty thousand and one.
24. Four hundred millions four thousand and forty.

---

[1] The learner is requested to test his ability to read numbers by trying to read the answers of the above examples.

ADDITION.

12. ADDITION is the process of finding one number equal to two or more. The result or answer is called the *Sum.* The numbers to be added are called *Addends.*

The addition of abstract numbers or of applicate numbers of one denomination, is called SIMPLE ADDITION.

When a number is not applied to any particular object it is called an *abstract* number. An applicate number is applied to some objects, as 10 men, 20 pounds, 9 shillings.

The *sign* of addition is +; thus 4 + 6, read, 4 *plus* or more 6, and means that 4 and 6 are to be added.

It is also necessary to explain the use of another sign, namely, =, read '*equal to*;' thus 4 + 6 = 10, read, *four plus six, equal to ten.*

The sign or abbreviation ∴ is frequently used for *therefore.*

13. RULE FOR SIMPLE ADDITION.

1°. Write the *addends* under each other, so that *units* may stand under *units.*

2°. Add up the figures in the *units' place:* set down the *units* of the sum and carry the tens, if any, to the next or tens' column; proceed in the same manner with the remaining columns.

3°. Having added the last column, set down its sum in full.

EXAMPLE 1. Find the sum of 424, 134, 327, and 26.

$$\left.\begin{array}{r}424\\134\\327\\26\end{array}\right\}\text{ addends.}$$

$$\begin{array}{r}\overline{911}\end{array}\text{ Sum.}$$

The numbers are written as in the annexed operation. Then commencing at the right or units' place, we say, 6 and 7 are 13; 13 and 4 are 17; 17 and 4 are 21 *units* or *ones,* which are = 2 tens and 1 unit: we set down the 1 *unit*; and *carrying* the 2 tens to the next column, we say, 2 and 2 are 4; 4 and 2 are 6; 6 and 3 are 9; 9 and 2 are 11 = 1 hundred and 1 ten; we set down the 1 *ten*, and *carry* the 1 hundred to the next column, the sum of which is 9; we set down the 9. The sum or answer is 911.

14. Reason of the Rule.—The rule for addition is obvious from the *principles of Notation* already given.

1°. If we place units under units, tens must be under tens, hundreds under hundreds, &c. We write the numbers in this order that we may have those figures of the *same kind* or *local value* under each other, since we cannot add *units* to *tens*, or *tens* to *any other column* except its own.

2°. We commence at the *units' place* for the convenience of *carrying* to the higher places. We set down the number of units in the sum of this column, and carry the tens to the tens' column, which is the same kind as what we carry, and so on with the other columns.

3°. We always write the sum of the *left hand* column in full, because we have nothing more to add to it.

15. Proof.—I. The most convenient mode of proving Addition is to begin at the top and count it down: if the two results agree, the work is *most probably* correct.

II. Another method of proof, depending upon the principle, 'The whole is equal to the sum of all its parts,' will be understood from the following example:—

In writing the example in order to prove it we *omit the first addend* 9864. Having found the sum of the remaining addends, 19123, we add the *neglected addend* to it, and thus find the same result, 28987. It is plain that the omission of any other addend would answer equally well. To avoid the necessity of writing the example *twice* as in the above explanation, it will be sufficient to draw a line under the *first addend* and omit this addend in the addition intended as proof.

Example.	*Proof.*
9864	4769
4769	7907
7907	5869
5869	578
578	19123
28987	9864
	28987

II.

Find the sum of:—

1.	2.	3.
39704	760987	9786549897
88888	859378	7654398799
76543	999999	6547998887
9799	75867	6437896467
3787	937899	7776666698

4. 4059 + 760786 + 439 + 986407 + 408067.

5. 54 + 960483 + 197 + 546078 + 88067 + 478609.

6. 980 + 43609 + 9908 + 78078 + 11011 + 999873.

7. 777 + 439 + 709053 + 54806 + 655073 + 864 + 1998.

8. Add, one hundred and seventy-nine; nine thousand four hundred and eighty-nine; six hundred and twelve thousand seven hundred and ninety-six; nine hundred and sixteen thousand seven hundred and forty; sixty-nine thousand seven hundred and eighty-seven.

9. Add, three hundred and sixty-five; four hundred and seventy-nine; six hundred and ninety-four; seven hundred and nine; nine hundred and seventy-eight; forty-eight; nineteen.

10. Find the sum of, four hundred and nine thousand seven hundred and forty-nine; six hundred and seventy-eight thousand and seventy-five; eight hundred and sixty-nine thousand and fifty; nine thousand eight hundred and seventy-six; five hundred and seventy-four.

11. Add, thirty-nine millions six hundred and ninety-four thousand four hundred and ten; forty millions forty thousand and forty; ninety-seven millions ninety-seven thousand and ninety-seven; fifty-six thousand and seven; three hundred and seventy-nine; five hundred and eight.

12. Adam lived 930 years; Seth, 912; Enos, 905; Cainan, 910; Mahaleel, 895; Jared, 962; Enoch, 365; Methusaleh, 969; Lamech, 777; Noah, 950; Shem, 600; Arphaxad, 438; Salah, 433; Heber, 464; Peleg, 239; Reu, 239; Serug, 230; Nahor, 148; Terah, 205; Abraham 175; Isaac, 180; Jacob, 147; Joseph, 110; Moses, 120; Joshua, 110. What is the sum of all their ages?

Post Office, Tidewaiters, &c.

13. 64935039
3945684
59712593
78916376
12863121
83752463

14. 64935039
3945684
59712593
80643978
12863121
83752463

15. 34689104
83674823
97615307
41923584
76325918
67431297
93206140
72169837

16. 34689104
83674823
97615307
41923584
13895193
67431297
93206140
72169837

Apprentices in Dockyards.

17. 65035049
3729015
72823652
5984315
38140238
94863574

18.* 367443
52421
1671
2596
924
34853
684
24835
87
72

Various Departments.

19. 75053079
344515
4477896
81390045
78963412
6547885

20. 64035039
3945684
59712593
78916376
12863121
83752463

Letter Carriers and Rural Messengers in Post Office (Time 15 *minutes.)*

21. 145
739
4639
504

22. 823
210
843
1525

23. 82
903
715
4786

24. 613
50
453
3709

The learner should be accustomed to add in the following manner until he can do so with accuracy and despatch: Because, 4 and 7 are 11, 14 and 7 are 21, 24 and 7 are 31. Again, 16 and 8 are 24; say, 'because 6 and 8 are 14,' 26 and 8 are 34, 36 and 8 are 44, and so on. It is necessary to observe that 11, 21, 31, &c., all end in the same figure.

* Set to tidewaiters.

SUBTRACTION.

16. SUBTRACTION is the process of finding the difference between any two numbers.

The number to be subtracted is called the *Subtrahend*; the number from which the *Subtrahend* is to be taken is called the *Minuend.* The result or answer is called the *Remainder* in commercial arithmetic, and the *Difference* in mathematical science.

SIMPLE SUBTRACTION, like SIMPLE ADDITION, is confined to *abstract* numbers or *applicate* numbers of but one denomination.

17. RULE FOR SIMPLE SUBTRACTION.

1°. Write the smaller number under the *greater*, just as two addends are written.

2°. Take *each* figure in the smaller number from the figure *corresponding* to it in the larger, commencing at the units' place.

3°. If the *lower* figure be greater than the figure *over* it, increase the *latter* by 10, and subtract. After setting down the remainder, *carry* 1 to the next figure to be subtracted, and so on.

Ex. 1. From 739 take 434.

In this example we say, 4 from 9 and 5 remain; 3 from 3 and nothing remain; 4 from 7 and 3 remain.

$$\begin{array}{r} 739 \\ 434 \\ \hline 305 \end{array} \quad \textit{Ans.}$$

Ex. 2. What is the difference between 7372 and 3454?

We cannot take 4 units from 2 units, then we say 4 from 12 (10 + 2) and 8 remain. *Carry* 1 to 5, 6 now, 6 from 7 and 1 remains; 4 from 3, we cannot, but 4 from 13 and 9 remain; carry 1 to 3 make 4, 4 from 7 and 3.

$$\begin{array}{r} 7372 \\ 3454 \\ \hline 3918 \end{array} \quad \textit{Ans.}$$

18. REASON.—The reason of the rule is obvious, and depends on the fact, that the difference of the *corresponding parts* of two numbers is equal to the *difference* of the numbers themselves. If we wish to take 59 from 161, we can say 161 = 150 + 11 and 59 = 50 + 9, hence we have

		Explanation.
161	=	150 + 11
59	=	50 + 9
102		100 + 2 = 102

19. Proof.—I. Add the remainder to the smaller number, and the sum will be equal to the greater.

II. Subtract the answer from the greater number, and the remainder will be equal to the less.

The sign of subtraction is —, thus, 10—6 = 4; read 10 minus or less 6 = 4, and denotes that the number after the sign is to be taken from the number which precedes it.

III.

	(1)	(2)	(3)	(4)
From	78069	86004	496736	69086
Take	16736	19363	279687	47979

5. 100100—3656.
6. 59064—56899.
7. 100300—7786.
8. 709600—46867.
9. 63712—16019.
10. 58634—57016.

Tidewaiters.

11. *Subtract* 868704368 *from* 7648200069.

Post Office.

12. *From* 683216432 *take* 89532179.
13. *From* 4723698007 *take* 494789127.
14. *From* 4723698007 *take* 493298127.

Dockyards.

15. *From* 84331306 *take* 56421808.

Various Departments.

16. *From* 256714894 *take* 93553760.

MULTIPLICATION.

20. Multiplication is a *short* method of *Addition* when the addends are all alike.

When the addends are all the same it is only necessary to set down one of them, now called the *Multiplicand*, or number to be multiplied.

The number showing how many addends are to be taken is called the *Multiplier.* The answer, or what is called the *Sum* in Addition, is called in Multiplication the ***Product.***

The following example will sufficiently illustrate these definitions.

Ex. 1. Multiply 7 by 4, in other words, add 7 four times.

By Addition, **7 + 7 + 7 + 7 = 28 *Ans.***

By Multiplication, **7 × 4 (read, 7 multiplied by 4) = 28. 7 is *the multiplicand,* 4 the *multiplier,* and 28 the *product.***

Proof.

. . . .
. . . .
. . . .
. . . .

The numbers 7 and 4 which are multiplied together are also called *factors,* in reference to the product.

The *sign* × implies that the numbers between which it is placed are to be multiplied together.

The multiplier and multiplicand may *change* places, without affecting the product, since 7 times 4 = 4 times 7.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Simple Multiplication is confined to the same kind of quantities as Simple Addition.

The multiplier must be invariably an abstract number, as it always indicates the number of times the multiplicand is to be taken.

Case I.

21. *When the multiplier is not greater than* 12.

Rule.—Multiply each figure of the multiplicand by the multiplier, by means of the *Multiplication Table*, setting down and carrying, just as in Addition.

Ex. 1. Multiply 3798 by 9.

$$\begin{array}{r} 3798 \\ 9 \\ \hline 34182 \end{array} \textit{ Ans.}$$

In the annexed example, we say, 9 times 8 = 72, set down 2, and *carry* 7; 9 times 9 = 81, and 7 = 88, write down 8, and *carry* 8, and so on. This case may be conveniently *proved* by Addition.

IV.

1. 98677 × 7.
2. 78069 × 9.
3. 86099 × 11.
4. 97645 × 12.
5. 70869 × 8.
6. 790063 × 11.

Case II.

22. *When the multiplier is greater than* 12.

Rule.—1°. Set down the multiplier under the multiplicand, just as two addends are written.

2°. Commencing with the units' figure, multiply every figure of the multiplicand by every figure of the multiplier, *taking care* to write the *first figure* of each product *as far* from the *units' place* as the *figure* of the multiplier which produces it.

3°. Add all these products, and their sum will be the complete product.

Ex. 2. Multiply 429 by 45, and 3736 by 127.

We first multiply by 5, as in Case I. Next we multiply by 4, placing the first figure, 6, of its product under the tens' place, as directed by the rule.

$$\begin{array}{rl} 429 & \\ 45 & \\ \hline 2145 & =\textit{prod. by } 5 \\ 17160 & =\textit{prod. by } 40 \\ \hline 19305 & =\textit{prod. by } 45 \end{array}$$

$$\begin{array}{rl} 3736 & \\ 127 & \\ \hline 26152 & =\textit{prod. by } 7 \\ 7472 & =\textit{prod. by } 20 \\ 3736 & =\textit{prod. by } 100 \\ \hline 474472 & =\textit{prod. by } 127 \end{array}$$

Ex. 3. Multiply 897063 by 504017.

$$\begin{array}{rl} 897063 & \\ 504017 & \\ \hline 6279441 & =\textit{product by } 7 \\ 897063 & =\textit{product by } 10 \\ 3588252 & =\textit{product by } 4000 \\ 4485315 & =\textit{product by } 500000 \\ \hline 452135002071 & = \textit{Total product, by } 504017 \end{array}$$

REASON.—

1°. This is a matter of convenience.

2°. Since we cannot multiply by the whole multiplier *at once*, we divide it into parts by taking each of its figures separately.

3°. This is self-evident.

PROOFS.—I. Multiply the multiplier by the multiplicand and the same result will be obtained.

II. The explanation given (**20**) is the simplest proof.

III. By *Addition*, as in (**20**).

IV. The most natural method of proof is by division; but this would be anticipating that rule, of which the learner is supposed to be ignorant.

V. If we commence with the left-hand figure of the multiplier and fall back to the right, instead of to the left, it will afford a method of proof. This is the eastern mode of multiplying.

VI. Cast the nines out of the sum of the figures of the multiplier and set down the remainder; cast the nines also out of the sum of the multiplicand and set down the remainder; multiply these two remainders together and cast the nines out of the product, this last remainder is the same as the remainder obtained by casting the nines out of the product if the work be correct.

In the second example, we find the sum of the figures of the multiplier, 4+5=9=*one* nine without any remainder; we write 0 above, as in the margin. Again, 4+2+9=15=*one* nine and 6 remainder; set down 6 opposite the first remainder 0. Now multiply these remainders, which gives 0. Lastly the sum of the figures in the product, 1+9+3+5=18=*two* nines, and leaving no remainder as before; we write 0 opposite the last figure written.

$$\begin{array}{ccc} & 0 & \\ 0 & & 0 \\ & 6 & \end{array}$$

IV.

7. 8679 × 23, and by 28.
8. 76979 × 48, „ 56.
9. 99706 × 73, „ 85.
10. 860478 × 95, „ 102.
11. 860978 × 127, „ 135.
12. 98653 × 557, „ 629.
13. 77869 × 560, „ 978.
14. 75060 × 914, „ 970.
15. 937860 × 9600, and by 908.
16. 207019 × 19307, „ 390.
17. 20136720 × 215670.
18. 40707900 × 6048070.
19. 9940806 × 9773060.
20. 7048960 × 7040980.
21. 9607587 × 948060.

CONTRACTIONS.

23. *When one or both of the factors end in cyphers.*

RULE.—Multiply as if there were *none*, and add so many cyphers to the result as have been neglected.

Ex. 4. 7200 × 3600. Here 72 × 36 = 2692, but we neglected *four* cyphers; therefore the *true* product is 26920000. Any cyphers in the multiplier except at the end are always neglected in the operation.

22. 86400 × 8700, and by 1700. 23. 19900 × 7600, and by 6500.

24. *To multiply by a composite number.*

RULE.—Multiply *successively* by its factors.

Ex. 5. 9648 × 42.

It is evident that 6 times the product by 7, will be the product by 42.

$$\begin{array}{rl} 9648 & \\ 7 & \\ \hline 67536 & = \textit{product by } 7 \\ 6 & \\ \hline 405216 & = \textit{product by } 42 \end{array}$$

24. 86736 × 64, and by 72. 25. 87697 × 121, and by 132.

25. *To multiply by any number consisting of nines.*

RULE.—Add as many cyphers to the multiplicand as there are nines in the multiplier, and subtract the *multiplicand* from the result. Thus, 864 × 99 = 86400 — 864 = 86366 *Ans.*

REASON.—When we add as many cyphers as there are nines we multiply by a number 1 *too great*, hence we must subtract the multiplicand. For, 100 = 99 + 1; 1000 = 999 + 1; and 10 = 9 + 1, &c.

26. Always in multiplying take the *most convenient* number as multiplier. Thus, 7 × 112, say, 7 times 2, &c. This *hint* will be found of considerable utility in many subsequent operations.

IV.

Post Office.

26. Multiply 60538067 by 83.
27. " 64059008 by 720106.
28. " 60538067 by 94.
29. " 64059008 by 807609.
30. " 46054 by 78.
31. " 247003 by 6020.
32. " 46054 by 68.
33. " 247003 by 4050.

Dockyards.

34. Multiply 96072015 by 2603.

Various Departments.

35. Multiply 98703542 by 700706.

Tidewaiter and Boatman.

36. Multiply 978015632 by 9738.

When more than two factors are employed in forming a product, it is called a *Continued Product.* Thus, 3 × 4 × 6 = 72, the continued product of 3, 4, and 6.

NOTES.—1. The method of *proof* (vi.) depends on a property of the number 9, viz.:—'any number divided by 9 will leave the same remainder as the sum of its figures divided by 9.'

2. The explanation of the annexed example in multiplication will be found useful.

Having multiplied by 1 (written the multiplicand), multiply *this product* by the next figure 2, because it is more directly under the eye than the multiplicand. The product by 4 should be found by taking twice the product by 2; the product by 8, by taking twice this last product. For the product by 4, the left-hand figure of the multiplier, write the product by 4 already found, or, take one-half of the product by 8.

```
     763479
      48421
     ------
     763479
    1526958
   3053916
  6107832
 3053916
-----------
36968416659
```

3. When the figures of the multiplicand are fewer than those of the multiplier, or afford facilities for multiplying, like those pointed out above (the multiplier not having similar advantages), multiply as in **26**.

DIVISION.

27. DIVISION is the process of finding how often one number is contained in another. The number to be divided is called the *Dividend*; the number by which the *dividend* is to be divided, is called the *Divisor*; and the result or answer the *Quotient*.

When the dividend does not contain the divisor an *exact* number of times, there is a *Remainder*, which is always less than the divisor.

DIVISION enables us to find the *third* part, or *any* part, of a number; or, it shows how to find one of the factors of a given product, when the other is known.

As this rule shows how often one number must be taken to make up another, it is plain that the number of times we can *subtract* a less number from a greater, is the number of times the greater contains the less. Therefore Division is a *contracted* mode of *Subtraction*, and bears a relation to it somewhat *similar* to that which *Multiplication* has to *Addition*. For instance, if we require to know how often 5 is contained in 30, we may say, $30-5=25$, and $25-5=20$, and so on, until the remainder becomes 0, or *less* than 5; the number of times we subtract is the *Quotient*. But we see at once that 30 contains 5, 6 times by the MULTIPLICATION TABLE.

The *sign* of Division is ÷, and indicates, when placed between two numbers, that the number which precedes is to be divided by that which follows it. DIVISION is also expressed by writing the divisor *under* the dividend, separated by a line. Thus $40\div10$, and $\frac{40}{10}$, mean the *same* thing; read, 40 divided by 10.—($40\div10=4$).

CASE I.

28. *When the divisor does not exceed* 12.

Rule.—1°. Set down the divisor to the *left* of the dividend, separating them by a *curve* line.

2°. Find by the Multiplication Table how often the divisor is contained in the *first* figure, or, if necessary, the number expressed by the first *two* or the first *three* figures of the dividend; set down the figure denoting the number of times as the first figure of the quotient. Subtract *mentally*, the product of this quotient figure and the divisor, from the number expressed by the figures of the dividend already used.

3°. To the remainder annex the next figure of the dividend and proceed as before, until all the figures of the dividend have been considered.

4°. Should there still be a remainder it must be written after the quotient and separated from it by +.

Ex. 1. Divide 3484 by 4.

Here we see that 4 will not be contained in 3, that is, we cannot take 4 from 3; therefore, we say, 4 into 34, 8 times and 2 over; we say 2 before 8 makes 28, 4 in 28, 7 times; and 4 into 4, once. Therefore the quotient is 871.

$$\begin{array}{r} 4)3484 \\ \hline 871 \text{ Ans.} \end{array}$$

Reason.—1°. Writing the dividend and divisor as directed is a matter of convenience.

2°. In the example, we require to take the *fourth* part of 3484: we cannot take the 4th-part part of 3000, since 3 will not contain 4; therefore we consider 3000 as 30 hundreds, to which when 400 is added it makes 3400; 4 will be contained in 34, 8 times, that is, 800 times, and leaves 2, that is 200. This last we consider not as 200, but as its equivalent 20 tens, to which we add 8 tens, making 28 tens, 4 into 28 tens, 7 tens times. Lastly, 4 into 4 once.

$$\begin{array}{rrr} 4) & 3484 & \\ & 3200 & 800 \\ \cline{2-2} 4) & 284 & 70 \\ & 280 & \\ \cline{2-2} 4) & 4 & 1 \\ & 4 & \\ \cline{2-3} & & 871 \text{ Ans.} \end{array}$$

Since $3484=3000+400+80+4$, the work in full will be as in margin.

V.

1. 98656÷3, and by 5. 2. 83295÷7, and by 8.

3. 9349344÷9, and by 12.

LONG DIVISION.

29. *When the divisor is greater than* 12.

RULE.—Instead of multiplying and subtracting *mentally* as in *Short Division*, we must write down the *whole* of the work as in the following example.

Ex. 2. 86560÷11.

As this is an example coming under *Short Division*, the best manner to understand the *reason* and *nature* of the process is to work the example both ways, and compare the different steps of each operation. We just say, 11 into 86, as in the former case, 7, then 7 × 11 = 77, which we write *under* 86 and subtract; to the remainder 9 we *bring down* 5, the next figure of the dividend, marking it by a dot to show that it has been *used*, and so on. After *all* the figures of the dividend have been brought down, there is a remainder 9, which is written as before. When any dividend, formed of a remainder and a figure brought down, is *less* than the divisor, we must place a 0 in the quotient and bring down another figure.

```
11)86568
   77···
   ——  (7869 + 9 Ans.
    95
    88
   ——
     76
     66
    ——
     108
      99
     ——
       9
```

Ex. 3. 1104654+327.

Here, as 3 will not be contained in 1, we must say 3 in 11, 3 times; we write 3 in the quotient and multiply. After subtracting, we say 3 in 12, 4 times, but 4 × 2 (the second figure of the divisor) = 8, which would be *too great*; therefore the proper quotient figure is 3. Again, 3 into 25, 8 times, but 8 × 2 = 16, which would also be *too great*, then we must put 7 in the quotient, &c.

```
327)1104654
    981···
    ————(3378 + 48 Ans.
    1236
     981
    ————
     2555
     2289
     ————
      2664
      2616
      ————
        48
```

PROOF.—I. Multiply the quotient by the divisor and the product + the remainder (if any), will be equal to the dividend.

II.—Subtract the remainder from the dividend, and divide what is left by the quotient; the new result or quotient will be the same as the given divisor.

V.

4. 70633 ÷ 23, and by 24.
5. 96802 ÷ 29, „ 37.
6. 186879 ÷ 63, „ 73.
7. 897608 ÷ 98, „ 106.
8. 2760865 ÷ 179, and by 224.
9. 6960508 ÷ 586, „ 625.
10. 864739 ÷ 907, „ 943.

Post Office, Customs, &c.—Junior Appointments.

11. Divide 7461968 by 8.
12. „ 7461968 by 9.
13. „ 434589760126 by 7.

Education Office.

14. Divide 106397320524 by 12.

III.—Add every *alternate* line of figures just as they stand in the operation, their *sum* should be equal to the dividend.

IV. Cast the *nines* out of the divisor and quotient, multiply the remainders and cast the nines out of their product; to this last remainder add the remainder found by casting the nines out of the remainder (if any) in the example. This sum should be equal to the remainder found by casting the nines out of the dividend.

REASON OF PROOFS.—I. This follows at once from the definition of DIVISION.

II. This is merely *reversing* the process.

III. This proof depends on the principle, 'The whole is equal to the sum of all its parts.'

IV. This depends upon a property of the number 9 already referred to.

30. As we are unable to see at once how often the divisor is contained in the dividend, we actually separate the latter into *parts* during the process, and find how often the divisor is contained in each part. The quotient is the *sum* of the number of times the divisor is contained in each part of the dividend. The dividend is divided in the following manner during the process:—*one part* is the product of the divisor, and the first quotient figure having its local value; the *other part* is the difference between this product and the whole dividend, and so on.

Whenever a figure is written in the quotient the same as one already placed there, write down the product of the former figure and divisor. If 3 be placed in the quotient after 6, take *one-half* of the product by 6, and so on.

31. The following *axioms* have been found useful, and may be employed with considerable advantage whenever they are applicable to an example: indeed, almost all the contractions in arithmetic are derived from these two simple principles.

I. The product of any two numbers is the *same* as *one-half* of one number multiplied by *twice* the other; or, *one-fourth* of one multiplied by *four times* the other, and so on. For instance, 380 yds. at 7½*d*. per yard = 190 at 1*s*. 3*d*. (7½*d*. × 2) = 95 at 2*s*. 6*d*., then take ½ = £11 17*s*. 6*d*. *Ans.*

Again 7½ yds. at 3*s*. 6*d*. = 15 at 1*s*. 9*d*. = 30 at 10½*d*. = 315*d*. = £1 6*s*. 3*d*. *Ans.* Also, 7 cwts. 2 qrs. 14 lbs. at £4 10*s*. 8*d*. = 15 cwts. 1 qr. at £2 5*s*. 4*d*. = 61 cwts. at 11*s*. 4*d*. = £34 11*s*. 4*d*. *Ans.*

II. The quotient of any two numbers is the same as *one-half* of one, divided by *one-half* the other, or *one-fourth* of one, divided by *one-fourth* of the other. It is also the same as *twice* one divided by *twice* the other, or any *multiple* or *part* of one, divided by the *same multiple, or a like part* of the other. For example, 18 ÷ 4½ = 36 ÷ 9 = 4; and 23⅓ ÷ 3⅓ = 70 ÷ 10 = 7; again, 1000 ÷ 68 = 500 ÷ 34 = 250 ÷ 17 = 14$\frac{12}{17}$.

To Candidates who are to know 'Elementary Arithmetic' are proposed two questions, such as the two following:—

15. Divide 846123998776 by 7.
16. „ 630762540981 by 652.

Admiralty (Dockyards).

17. Divide 750324098176 by 826.

Various Departments.

18. Divide 2370056 by 93.
19. „ 327840937575 by 716.
20. „ 5946784379 by 492.
21. „ 2370056 by 79.
22. „ 32780937575 by 589.

32. *To divide by a composite number.*

Rule.—1°. Divide first by one factor, as in Short Division, then divide this quotient by the other factor.

2°. Multiply the *last remainder* by the *first divisor*, and to the *product* add the *first remainder* for the *true* remainder.

Ex. 4. 86497 ÷ 36.

When we divide by the second factor 6, we have a remainder 4; this 4 is not 4 units but 4 *sixes*. Then 6 × 4 + 1 = 25 the true remainder.

$$36\left\{\begin{array}{l}6)\underline{86497}\\6)\underline{14416}+1\end{array}\right.$$
$$2402+(4\times6+1)25.\ \textit{Ans.}$$

V.

23. 486906 ÷ 42, and by 81.
24. 786456 ÷ 108, and by 121.
25. 5604060 ÷ 385 (11 × 7 × 5), and by 48.
26. 7936497 ÷ 648 (9 × 8 × 9), and by 49.
27. 8695687 ÷ 792 (11 × 9 × 8), and by 108.

33. *When the divisor ends in cyphers.*

Rule.—1°. Neglect the cyphers, and cut off as many figures from the right of the dividend.

2°. Divide with the remaining figures of the divisor, and annex the figures cut off from the dividend to the remainder.

Ex. 5. 96846 ÷ 1200 = 968,46 ÷ 12,00 = 80 + 846 *Ans.*

Hence, to divide by any number consisting of 1 followed by any number of cyphers, we have merely to cut off from the right of the dividend as many figures as there are cyphers: the other figures of the dividend form the quotient, and those cut off, the remainder.

Ex. 6. 18767 ÷ 100 = 187,67 = 187 + 67 *Ans.*

Ex. 7. 96186 ÷ 7300.

```
73,00)961,86
      73
      —— (13 + 1286 Ans.
      231
      219
      ———
       1286
```

V.

28. 684379 ÷ 1400, and by 1000.
29. 469786 ÷ 100, and by 6300.
30. 9860400 ÷ 1700, and by 1860.

CONTRACTIONS.

34. The work of division may be considerably *contracted* by multiplying and subtracting *mentally* at the same time, *omitting* the product of the divisor and each quotient figure, and writing the remainder only. This method should not be introduced until the learner can work any example in the rule with the *greatest* ease; otherwise his progress will be *retarded.* The work will stand as below. Compare both methods, and the reason will be evident.

Usual Method.

```
173)6847329
    519····
    ——— (39579
    1657
    1557
    ————
     1003
      865
     ————
      1382
      1211
      ————
       1719
       1557
       ————
        162
```

Contracted Method.

```
173)6847329
    ———  ···
    1657 (39579
    ————
     1003
     ————
      1382
      ————
       1719
       ————
        162
```

35. *To divide by* 25.

Rule.—Multiply the dividend by 4; cut off *two* figures to the right; one fourth of the number expressed by these two figures will be the *true* remainder.

Ex. 8. 7324÷25=7324×4=29296, and 292|96=292+24 *Ans.*

36. *To multiply by* 25.

Rule.—Annex 00 and divide by 4.

Ex. 9. 375×25=37500÷4=9375 *Ans.*

37. *To divide by* 35, 45, 55, *&c*

Rule.—Divide *twice* the dividend by 70, 90, 110, &c. Take *half* the remainder for the *true* one.

Ex. 10. 69475÷35=138950÷70=1985 *Ans.*; and 785675÷45 =785675×2÷90=1571350÷90=17459+40=17459+20 *Ans.*

38. *To multiply by* 125.

Rule.—Annex 000 and divide by 8.

Ex. 11. 116×125=116000÷8=14500 *Ans.*

39. *To divide by* 125.

Rule.—Divide 8 times the dividend by 1000, and take *one-eighth* of the remainder for the *true* one.

Ex. 12. 2125÷125=2125×8÷1000=17000÷1000=17 *Ans.*

40. *To multiply by* 35, 45, 55, *&c.*

Rule.—Multiply by 70, 90, 110, &c., *respectively*, and take *one-half* of the product.

Ex. 13. 735×35=735×70÷2=51450÷2=25725 *Ans.*

V.

31. 968645÷25, and by 35.
32. 897895÷60, and by 125.
33. 978685÷25, and by 45.
34. 996846÷25, and by 45.

41. *To multiply when the multiplier contains* ONE-HALF *or* QUARTERS.

Rule.—For ½, add *one-half* of the multiplicand to the result obtained by the rule; for ¼, add *one-fourth* of it; and for ¾, add the result obtained by multiplying the multiplicand by 3 and dividing the product by 4.

Ex. 14. Multiply 168 by 7½.

Here we multiply 168 by 7 in the *usual* way; we then say 2 into 16, 8 times; 2 into 8, 4 times, and add the 84 to the product by 7.

$$\begin{array}{r}168\\ 7\tfrac{1}{2}\\ \hline 1176\\ 84\\ \hline 1260\ \textit{Ans.}\\ \hline\end{array}$$

Reason.—When we multiply any number by 1, we get that number itself as product, and of course when we multiply by ½, that is ½ of 1, we will get *one-half* the number as product.

V.

35. 97688 × 19½, and by 36½.

36. 86408 × 46¼, and by 86¾.

42. *To divide by any number containing* ONE-HALF *or* QUARTERS.

Rule.—1°. Reduce both dividend and divisor to *halves* or *quarters*; then use *these* as dividend and divisor.

2°. The remainder will be in *halves*, if we reduce to halves; in *quarters*, if we reduce to quarters.

Ex. 15. 444÷18½.

Here we reduce both to *halves*, and then divide.

$$\begin{array}{rrl}18\tfrac{1}{2})&444&\\ 2&2&\\ \hline 37)&888&\\ &74&\\ \cline{2-2} &&(24\ \textit{Ans.}\\ &148&\\ &148&\\ \cline{2-2}\end{array}$$

37. 86976÷9½, and by 25½.

38. 79684¼÷6¾, and by 29¼.

Note.—In any example in division we may divide the divisor and dividend by the same number, and use the quotients instead of the original numbers. Thus, 96÷20 = 24÷5 = 4⅘.

COMPOUND ARITHMETIC.

Weights, Measures, and Tables.

I. Money.[*]

4 farthings = 1 penny
12 pence = 1 shilling
24*d.* or 2*s.* = 1 florin
30*d.* = 1 half-crown
60*d.* or 2 h. cr. = 1 crown
8h. crs. or 10f. or 20*s.* = £1
21 shillings = 1 guinea

The *symbols* for pounds, shillings, and pence, are, *£*, *s*, *d.*

II. Avoirdupois.[†]

16 drachms = 1 ounce
16 ounces = 1 pound
14 pounds = 1 stone
28 pounds or 2 st. = 1 quarter
4 quarters or 8 st. or 112 lbs. = 1 hundredwt.
20 hundreds or 160 st. = 1 ton

The *symbols* respectively are, dr., oz., lb., st., qr., cwt., t.

III. Troy.

24 grains = 1 pennywt.
20 pennywts. = 1 ounce
12 ounces = 1 pound

The *symbols* are, gr., dwt., oz., lb. This weight is used for *gold*, *silver*, and *jewellery*; and in *philosophical experiments*. The pound *troy* contains 5760 grains, the pound *avoirdupois* 7000 grains.

IV. Apothecaries.

20 grains = 1 scruple
3 scruples = 1 drachm
8 drachms = 1 ounce
12 ounces = 1 pound

The *symbols* are, gr., sc., dr., oz. Apothecaries use it for *mixing* their medicines. The *grain*, *ounce*, and *pound*, are the *same* as in Troy.

---

[*] Our *standard* gold coins consist of 22 parts pure gold, and 2 parts copper or alloy; that is, $\frac{11}{12}$ pure gold and $\frac{1}{12}$ copper. The value of gold, from which the coins are made, called gold of the *mint fineness*, is £3 17*s.* 10½*d.* per ounce. From every pound Troy of standard gold, 46$\frac{29}{40}$ sovereigns are made; hence every sovereign weighs 127·2744 grains. Gold is a *legal tender*, without limitation, and is the *general standard* of value.

Our *standard* silver coins are made of a metal consisting of 37 parts of pure silver and 3 parts copper. The *mint price* of standard silver is 5*s.* 6*d.* per ounce. Every pound Troy of silver is coined into 66*s.*, consequently a shilling weighs 87·2727 grains. Silver is a *legal tender* for any sum not exceeding £2.

Copper is now coined at the rate of 48 pence to the pound Avoirdupois; hence a penny weighs $\frac{1}{3}$ of an ounce, and 12 pennies $\frac{1}{4}$ of a pound. Copper is a *legal tender* only to the amount of one shilling.

[†] The *imperial pound*, Avoirdupois, is the unit of weight for all heavy goods. An *imperial gallon of distilled* water weighs 10lb., and consists of 277·274 cubic inches. A

V. CLOTH.

2¼ inches = 1 nail
4 nails = 1 quarter
4 quarters = 1 yard
5 quarters = 1 ell, English
3 quarters = 1 Flemish ell
6 quarters = 1 French ell

VI. LENGTH.

12 inches = 1 foot
3 feet = 1 yard
5½ yards = 1 pole or perch
40 poles = 1 furlong
8 furlongs = 1 mile
3 miles = 1 league
7 yards = 1 Irish perch

VII. GRAIN.

6 stones = 1 barrel of *bran*
12 „ = 1 „ of *malt*
14 „ = 1 „ of *oats*
16 „ = 1 „ of *bere*
20 „ = 1 „ of *peas* or *wheat*
16 „ = 1 *bag* of flour
20 „ = 1 *sack* of flour
2 barrels = 1 *quarter* of wheat or malt

VIII. SURFACE.*

144 *square* in. = 1 *square* foot
9 *square* feet = 1 *square* yard
30¼ *sq*. yds. = 1 *square* rod, pole, or perch
40 perches = 1 rood
4 roods = 1 acre
49 *sq*. yds. = 1 sq. perch, *Irish*

IX. CAPACITY.

4 gills or naggins = 1 pint
2 pints = 1 quart
2 quarts = 1 pottle
2 pottles = 1 gallon
2 gallons = 1 peck
4 pecks = 1 bushel
8 bushels = 1 quarter
5 quarters = 1 load
63 gallons = 1 hogshead
36 gallons = 1 barrel of beer
2 hogsheads = 1 pipe or butt
2 pipes = 1 tun

In some places a barrel of beer has 32, in others, 34 gallons.

X. TIME.

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
4 weeks = 1 month
12 months = 1 year
365 days = 1 year
366 days = 1 *leap year*

A *leap-year* occurs every fourth year. If the number expressing the year can be divided by 4 without leaving a remainder, it is a leap-year, except it be an exact century.

XI. SOLIDITY.

1728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard
40 cubic ft. rough timber } 1 ton
50 cubic ft. hewn timber } or load
42 cubic ft. = 1 ton of shipping

---

pound, *Avoirdupois*, is = 14 oz. 11 dwt. 16 gr. *Troy*: therefore an ounce Troy, *exceeds* an ounce Avoirdupois by 1 dwt. 18½ gr. 144 lbs. Avoirdupois = 175 lbs. Troy.

* The *difference* between Irish and English measure is in the *perch*: 5½ yds. make 1 p. English, and 7 yds. 1 p. Irish; hence 11 Irish miles = 14 English miles, and since $11^2 = 121$ and $14^2 = 196$, it follows, that 121 Irish acres = 196 English acres, or 30¼ Irish = 49 English acres, or 3 a. 3 r. 5 p. Irish = 6 a. 0 r. 20 p. English.

XII. Surveyors' Measure.

100 links or 4 perches = 1 chain
10 chains = 1 acre
1 mile = 80 chains
1 square mile = 640 acres

The Circle

Consists of 360 degrees, each degree = 60 minutes, and each minute = 60 seconds. Degrees, minutes, and seconds are marked thus, °, ′, ″. Every degree is = 60 geographical or $69\frac{1}{10}$ English miles.

XIII. Coals.

5 stones = 1 bushel
2 bushels = 1 bag
8 „ = 1 quarter
2 bags = 1 barrel
8 barrels = 1 ton

Various.

12 articles = 1 dozen
12 dozen = 1 gross
20 articles = 1 score
5 score = 1 hundred
6 score = 1 long hund.
24 shts. of paper = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
30 deals = 1 quarter of timber.
120 deals = 1 great hundred.

The Civil Service Commissioners usually examine Tidewaiters, Weighers, &c., on the Tables, thus:—

'Write out the tables of

Avoirdupois weight, Troy weight, Long measure, Square measure, Wine measure, Ale and Beer measure.'

The following have also been asked:—

'Write out—

Measure of Time.
Dry Measure.
Ale and Beer Measure.'

VI.

Miscellaneous Questions.

1. Multiply ninety thousand seven hundred and eight by sixteen thousand and thirteen, and add 99 to the result.

2. From the product of thirteen thousand and nine and nine thousand and sixteen, take nine hundred and ninety-eight.

3. Divide the sum of 7896 and 8908 by 11.

4. Multiply the difference of 1000 and 53 by 79.

5. To the product of 1013 and 7009 add 73, and divide the sum by 9.

6. Take sixty thousand and fifty-four from seventy-eight thousand and ninety, and multiply the remainder by seven hundred and sixty.

7. From the product of one thousand and one hundred and three, take 99.

8. From the quotient of thirteen thousand and thirty-nine by thirteen, take 4.

9. Write down the sum, difference, product, and quotient of 9603 and 1067.

10. The product of two numbers, one of which is 365, is 395455. what is the other ?

11. What is the continued product of the sum, difference, product, and quotient of 72 and 24 ?

12. The quotient of two numbers, the lesser of which is 49, is 347, and the remainder is 27 ; what is the greater ?

13. How many days were in the first five months of the year 1860 ?

14. How many days from January 20, 1857, until August 14, 1863 ?

15. A regiment was supplied with 576 horses which cost £25 each ; what was the entire cost ?

16. How many pages in 30294 words, every page having 51 lines of 9 words each ?

17. How many farthings are there in £497, there being 960 in a £ ?

18. How many pence should a person pay for 44 lbs. of beef at 8¾*d.* per lb. ?

19. How many chickens at 10½*d.* each could a person buy for 115½*d.* ?

20. Divide 25*s.* between two boys, giving one 3*s.* more than the other.

21. What is the number of lines in the page of a book which contains 1110 letters, every 37 letters forming a line ?

22. What is the difference between the continued product of 17, 93, 46, 59, and that of 227, 247, and 58 ?

23. Add together 379876, 583974, 3978653, 98, 7894, 569, 3747, 7, 974869, 89767, 54869, 37897, 9879, 5678089, 24, and 397, and take 11796986 from the sum.

24. Add all the numbers ending in 9 from 1 to 300 together.

Proposed to Tidewaiters and Weighers.

25. Multiply four millions forty-nine thousand and three, by thirty-six thousand and twenty-nine.

REDUCTION.

43. REDUCTION is the bringing of a quantity, consisting of one or more denominations, to another quantity of the same kind, without altering its value.

REDUCTION DESCENDING.

44. *To reduce a quantity to a lower denomination.*

RULE.—Multiply the highest denomination by the number which shows how many of the next lower denomination make 1 of it, adding in the number of that denomination (if any), and so on, until the required denomination is arrived at.

Ex. 1. Reduce £59 into pence.

Here we multiply £59 by 20, because there will be 20 *times* as many shillings as pounds. Again, we multiply the shillings, 1180, by 12, because there will be 12 *times* as many pence as shillings.

£59
20
1180 *shillings.*
12
14160 *pence. Ans.*

Ex. 2. Reduce 29 tons 17 cwts. 3 qrs. 27 lbs. 12 oz. into oz.

tons	cwts.	qrs.	lbs.	oz.
29	17	3	27	12

20
580 + 17 = 597 cwts.
4
2388 + 3 = 2391 qrs.
28
19128
4782
66948 + 27 =
66975 lbs.
16
1071600 + 12 = 1071612 oz.

Read the table of Avoirdupois Weight, and the reason of the annexed operation will be evident.

Ex. 3. Reduce 18 miles 5 fur. 30 per. 2 yds. 2 ft. 10 in. into inches.

```
     mls. f. p. yds. ft. in.
      18  5 30  2   2  10
       8
     ---
     149 furlongs.
      40
    ----
    5990 perches.
      5½
   -----
   29952
    2995
   -----
   32947 yards.
       3
   -----
   98843 feet.
      12
 -------
 1186126 inches.
```

In this example the furlongs, perches, &c. are added in when multiplying, and not as in example 2.

VII.

Post Office.

1. Reduce 15 guineas to farthings.
2. „ 15 miles 2 furlongs 36 perches 2 yards to feet.
3. „ 42 cwts. 3 qrs. 17 lbs. 6 oz. to oz.
4. „ 18 tons 17 cwts. 2 qrs. 1 lb. to lbs.

Admiralty (Dockyards).

5. Reduce 42 miles 5 fur. 13 poles to inches.

Candidates who are to know 'Elementary Arithmetic,' such as in Police Courts, Science and Art, Board of Trade, Fisheries, &c.

6. Reduce 3 tons 9 cwts. 2 qrs. 4 lbs. 6 oz. to oz.
7. „ 1 ton 4 cwts. 3 qrs. 2 lbs. 3 oz. to oz.
8. „ 16 tons 4 cwts. 3 qrs. 15 lbs. 13 oz. to oz.
9. „ 13 miles 5 fur. 9 poles 3 yds. to inches.
10. „ 42 miles 3 fur. 23 poles 3 yds. to inches.

Tidewaiters, &c.

11. Reduce 159 quarters of corn to quarts.
12. In 34 lbs. 10 oz. 12 dwts. 16 grs. troy how many grains?

India Board.

13. Reduce 3 acres 20 rods 12 sq. yards and 7 sq. feet to square feet.

War Office.

14. How many grains are there in 17 lbs. 3 oz. 7 dwts. ?

Education and Constabulary Offices.

15. Reduce 15 lbs. 10 oz. 14 grs. troy to grains.

Candidates required to know 'Vulgar and Decimal Fractions.'

16. Reduce 2 miles 1 fur. 12 pls. 1 ft. 8 in. to inches.
17. „ 3 wks. 4 days 5 hrs. 54 mins. to seconds.
18. „ 13 yds. 7 ft. 19 inches (square measure) to inches.
19. „ 496 sq. yds. 2 sq. ft. to inches.
20. How many feet are there in 18 miles 6 fur. 12 pls. 2 yds. ?
21. Reduce 17 lbs. 6 oz. 12 dwts. 7 grs. to grains.
22. „ 54 cwts. 3 qrs. 18 lbs. 14 oz. to drachms.

Various Departments.

23. Reduce £9 19*s*. 9¾*d*. to farthings.
24. „ 13 lbs. 7 oz. 16 grs. to grains.
25. „ 47 cwts. 1 qr. 16 lbs. 2 oz. to drachms.
26. „ £527 16*s*. 8½*d*. to farthings.
27. „ 375 cwts. 2 qrs. 1 st. 13 lbs. to ounces.
28. * In 11 miles 0 fur. 6 pls. 4 yds. how many feet ?
29. Reduce 14 lbs. 11 oz. 11 grs. to grains.
30. „ 42 wks. 8¼ hrs. 15½ mins. 5 secs. to seconds.
31. How many seconds are there in 35 days 8 hrs. 48 mins. 29 secs. ?

REDUCTION ASCENDING.

45. *To reduce a quantity to a higher denomination.*

RULE.—Divide the quantity by the number expressing how many of this lower denomination make *one* of the next higher denomination; and so on, step by step, until the required denomination is arrived at.

Remainders are always the same name as the dividends from which they are derived.

In REDUCTION ASCENDING we use division : in REDUCTION

---

* Proposed to Tidewaiters and Weighers.

DESCENDING we use multiplication. The *one case* proves the *other*.

Ex. 1. To reduce 7328 farthings to pounds.

4)7328 *farthings.*
12)1832 *pence.*
2,0)15,2 + 8
£7 12*s.* 8*d.* *Ans.*

Proof of Example 1.
Reduce £7 12*s.* 8*d.* to *farthings.*
20
152
12
1832
4
7328 *farthings.*

We first divide by 4, because for every 4 farthings there will be *one* penny. We divide these pence, 1832, by 12; because for every 12 pence there must be *one* shilling; and lastly, we divide these shillings by 20 to bring them to pounds, using the *contracted* method.

Ex. 2. Reduce 73816 drachms to cwts.

$$16\left\{\begin{array}{l}4)73816 \text{ drachms.}\\ 4)18454\end{array}\right.$$

$$16\left\{\begin{array}{l}4)4613 \text{ oz.} + 8\ (4 \times 2)\\ 4)1153 + 1\end{array}\right.$$

$$28\left\{\begin{array}{l}7)288 \text{ pounds} + 5\\ 4)41 + 1\end{array}\right.$$

4)10 qrs. + (1 × 7 + 1)8

2 cwts. 2 qrs. 8 lbs. 5oz. 8 drs. *Ans.*

Proof of Example 2.
Reduce 2 cwts. 2qrs. 8lbs. 5oz. 8drs. to drs.
4
10 quarters.
28
288 pounds.
16
4613 ounces.
16
73816 drachms.

As 16 drachms make 1 ounce, we must first divide by 16: we use its factors 4 × 4. Having reduced to ounces, we divide by 16 again, to bring these to pounds; next, we divide by 28, since 28 lbs. = 1 qr.; for this we use the factors 7 and 4; and lastly, we divide by 4, to bring the quarters to cwts., and find 2 cwts. 2 qrs. 8 lbs. 5 oz. 8 drs. *Ans.*

Ex. 3. In 3196800 cubic inches how many loads of hewn timber?

```
1728)3196800
     1728
     ―――― (1850 cubic feet.
     14688
     13824
     ―――――
       8640
       8640          Then 5,0)185,0
       ――――                ―――――
          0                 37 loads. Ans.
```

In this example we reduce the cubic inches to cubic feet, and find 1850, which divided by 50, the number of cubic feet in a load, gives 37, the answer.

VIII.

Post Office.

1. In 6169 pence how many pounds?
2. In 15346907 oz. how many tons, cwts., &c.?

India Board.

3. In 523769 grains (Troy weight) how many lbs., oz., &c.?
4. In 569705 sq. ft. how many acres, roods, &c.?
5. In 29336935 seconds how many weeks, days, &c.?
6. How many lbs., oz., &c., are there in 111454 grains of gold?
7. How many miles, furlongs, &c., are there in 348164 inches?

Excise.

8. In 457241 lbs. (Avoirdupois) how many tons, cwts., &c.?
9. In 640825 seconds how many weeks, days, &c.?
0. In 2160000 cubic inches how many loads of hewn timber?
11. In 150000 eggs how many gross?

Education and Constabulary Offices.

12. Reduce 34677 square yards to acres, &c.

Where Candidates should know 'Vulgar and Decimal Fractions.'

13. In 23221 grs. of gold how many lbs., oz., &c.?
14. In 32391 oz. of sugar how many tons, cwts., &c.?
15. In 5792685 inches how many miles?

Various Departments.

16. How many miles, &c., are there in 247391 inches?
17. How many lbs., oz., &c., are there in 266450 grs. (Troy)?
18. Reduce 657918 cubic inches to yards, feet, and inches.

19. Reduce 138297 cubic inches to yards.
20. How many miles are there in 3126749 inches?
21. In 86754 oz. (Avoirdupois) how many tons, &c.?
22. Reduce 986877 cubic inches to yards, feet, and inches.

COMPOUND REDUCTION.

46. In several exercises, both of the cases of Reduction already considered have to be employed, as in the following questions:

Various Examinations.

Ex. 1. How many half-crowns are there in 83654 dollars, each worth 4*s.* 6*d.*?

dols.
83654
54*d.* = 4*s.* 6*d.*

334616
418270

a half-crown = 3,0*d.*)451731,6*d.*

150577 *dols.* 6*d.*

Post Office.

23. How many fourpenny pieces in 37689*s.*?
24. In £2470 how many crowns?
25. How many half sovereigns in 7642 guineas?

Excise.

26. Reduce 564 English ells to nails.

Various Departments.

27. How many francs at 9½*d.* each are there in £1734 6*s.* 10½*d.*?

Not proposed at Examinations.

28. A person has 250 fourpenny pieces and 57 threepenny pieces, how much money has he?

29. How many labourers can a person employ with 35 guineas, each receiving half-a-crown?

COMPOUND ADDITION.

47. A Compound Number consists of quantities of the same kind, but of different denominations.

RULE.—1°. Set down the addends so that the same denominations may stand under each other.

2. Commencing at the right hand, or lowest denomination, add it up; find how many *ones* or *units* of the next higher denomination in the sum; set down the remainder (if any), and carry the *ones* to the next denomination, and so on.

Ex. 1. Add £67 16*s.* 8½*d.*; £100 13*s.* 7¾*d.*; £168 16*s.* 10½*d.* together.

We begin by adding the farthings, and find their sum 7*f.* = 1 penny and ¾: we write the ¾ under the farthings, and *carry* the 1*d.* to the pence. We find the sum of the pence column 26, and 26*d.* = 2*s.* 2*d.*, set down the 2*d.* and *carry* the 2*s.* In adding the shillings, we say, 2 and 6 are 8, 8 and 3 are 11, 11 and 6 are 17, set down 7 and carry 1. This 1 and the 3 *ones* in the tens' column make 4; we say half of 4 is two, and carry 2 to the pounds, because there will be half as many *pounds* as *tens* in the shillings. If the sum of the tens be odd we write down 1, and carry the half of the next smaller even number.

£	*s.*	*d.*
67	16	8½
100	13	7¾
168	16	10½
337	7	2¾

Ex. 2.—Add 148 cwts. 2 qrs. 16 lbs.; 96 cwts. 3 qrs. 22 lbs.; 149 cwts. 1 qr. 10 lbs.; 729 cwts. 3 qrs. 23 lbs. together.

In this example we add the column lbs. and find the sum 71. Now, as 28 lbs. = 1 qr., we divide 71 by 28, which gives 2 qrs. 15 lbs. quotient: we set down the 15 lbs. and carry 2. We find the sum of the quarters = 11; this divided by 4, gives 2 cwts. 3 qrs. We set down the 3 and *carry* 2, to the addition of the *cwts.*

cwts.	qrs.	lbs.
148	2	16
96	3	22
149	1	10
729	3	23
1124	3	15

The best way to add the *shillings' column* of an account is, to disregard the tens until the units have been added; then, having reached the top, add downwards, counting 10 for every 1 in the tens' column.

In the annexed example the sum of the units' column of shillings is found, 27; then proceeding downwards say, 27, 37, 47, 57, 67 shillings = £3 7*s.*

£	*s.*	*d.*
48	16	10½
56	19	7¾
84	12	10
78	18	6½
£269	7	10¾ *Ans.*

We may proceed in the same manner in adding the column of lbs., adding 20 for every 2 in the tens' place, and 10 for every 1. Say in Ex. 2, 3 + 2 + 6 = 11; then, adding downwards, say, 21, 41, 51, 71 lbs. = 2 qrs. 15 lbs. The perches' column may be *similarly* added.

IX.

1.

£	*s.*	*d.*
7396	19	11¾
8977	17	10½
973	18	4¼
36406	9	9
6787	17	11½
596	16	9¾
498	17	11½
9877	15	10¾

2.

£	*s.*	*d.*
9745	16	10½
7859	17	11¾
897	19	9¼
5788	18	6¾
799	9	11½
6849	18	10¼
769	13	11¾
9898	19	8½

3.

cwts.	qrs.	lbs.
696	3	24
523	1	16
196	3	17
567	3	18
167	1	25

4.

cwts.	qrs.	lbs.	oz.
69	3	14	10
58	2	23	13
68	3	10	12
99	3	19	13

5.

lbs.	oz.	dwts.
39	11	13
56	10	9
96	9	8
59	10	10
67	10	19

6.

lbs.	oz.	dwts.
89	10	16
56	9	19
89	11	13
56	10	17
80	11	11

7.

acrs.	r.	pls.
19	3	35
17	2	27
19	3	16
15	2	29
16	3	36

8.

acrs.	r.	pls.
96	2	29
10	3	36
16	2	27
19	3	30
14	2	37

	acrs.	r.	p.	yds.
9.	64	2	30	20
	67	3	37	30
	69	2	36	27
	56	3	35	19
	19	3	25	25

	acrs.	r.	p.	yds.
10.	67	3	30	23
	19	2	17	19
	17	3	39	29
	18	2	36	20
	19	3	19	17

	degs.	m.	secs.
11.	106	30	35
	97	50	45
	109	20	25
	25	25	55
	317	30	30

	degs.	m.	secs.
12.	179	30	35
	806	26	36
	798	30	40
	779	36	50
	500	50	55

	gals.	qt.	nag.
13.	20	3	3
	17	2	2
	19	1	1
	23	3	2
	24	2	3

	gals.	qt.	pt.
14.	23	2	1
	27	2	1
	26	3	1
	25	2	1
	27	3	0

Post Office (Letter-carriers and Mail-guards).

	£	s.	d.
15.	18	17	9
	16	11	4
	18	9	11
	82	14	6
	12	16	4
	4	3	7

	£	s.	d.
16.	18	17	9
	27	9	10
	9	17	4
	78	13	9
	93	15	7
	4	3	7

	£	s.	d.
17.	18	17	9
	27	9	10
	9	17	4
	64	15	8
	93	15	7
	4	3	7

	£	s.	d.
18.	21	19	8½
	67	11	7½
	7	2	5
	89	15	7¾
	36	13	7½
	24	3	7½

	£	s.	d.
19.	21	19	8½
	67	11	7½
	7	2	5
	76	17	8½
	36	13	7½
	24	3	7½

	£	s.	d.
20.	366	13	4¼
	78	19	2½
	942	16	8¾
	1268	17	6½
	2876	15	2
	5534	2	0

	£	s.	d.
21.	21	19	8½
	16	13	6¾
	7	2	5
	43	17	8¾
	63	2	6¼
	24	3	7½

	£	s.	d.
22.	458	10	11¼
	1087	2	3½
	942	16	7¼
	1234	5	6
	114	13	8
	29	19	11¾

Junior Appointments in War Department and Post Office, &c.

	£	s.	d.
23.	5873	13	10½
	634	17	6½
	906	9	2½
	1347	9	11¾
	325	7	6
	923	13	11½

	£	s.	d.
24.	3298	17	10
	634	17	6½
	906	9	2½
	1347	9	11¾
	325	7	6
	923	13	11½

	£	s.	d.
25.	7089	14	5½
	508	3	9
	2530	6	10½
	47	15	8¾
	3401	11	7
	9320	16	9¾

	£	s.	d.
26.	7089	14	5½
	508	3	9
	8403	4	2½
	47	15	8¾
	3401	11	7
	65	5	5½
	7019	12	10
	9320	16	9¾

Admiralty (Dockyards).

	£	s.	d.
7.	2187	16	9
	634	17	6½
	8164	16	2½
	113	5	2½
	5016	15	7½
	923	13	11½

	£	s.	d.
28.*	5873	13	10½
	634	17	6½
	906	9	2½
	1347	9	11¾
	325	7	6
	923	13	11½
	794	6	10
	39	17	10½

	£	s.	d.
29.*	2917	10	2½
	27	1	2½
	533	7	6¼
	4456	8	9
	5001	17	8¾
	7	8	9

Tidewaiter and Boatman.

	£	s.	d.
30.	912	17	8
	5	10	9¾
	79	13	10¼
	365	9	5½

	tons	cwts.	qrs.	lbs.
31.	34	16	3	18
	1	19	2	14
	56	13	3	22
	19	18	2	16

---

* Proposed to candidates required to know '*Elementary Arithmetic*.'

	yds.	ft.	in.
32.	19	10	7
	3	7	9¾
	12	8	10½
	7	11	8¼

	hhds.	gals.	qrts.	pts.
33.	52	38	3	2
	19	58	3	1
	7	29	3	2
	85	48	2	1

Minor Situations in Post Office.

	mls.	fur.	per.	yds.
34.	37	3	14	2
	28	14	16	2
	19	10	9	4
	10	5	6	3
	20	2	2	2
	30	1	1	4

	mls.	fur.	pls.
35.	12	2	39
	15	1	19
	17	1	10
	19	2	15
	14	2	16
	122	4	2

	hrs.	min.	sec.
36.	20	37	40
	17	20	35
	21	16	34
	16	27	46
	22	19	52
	19	22	16
	21	31	37

	yrs.	mnths.	wks.	dys.
37.	76	8	3	6
	57	11	2	3
	34	9	3	5
	57	6	1	2
	35	10	2	4
	56	9	3	3
	20	6	1	2

Add up as many of the columns as you can in the time allowed (¾ of an hour), placing the answers in the spaces below the columns.

It is important the addition should be quite *correct*; additional credit will also be given for *rapidity*.

In some examinations, which are not competitive, only six columns are given, the time being half an hour. In others only 3 columns are given.

India Office.

	£	s.	d.		£	s.	d.		£	s.	d.
3.	134	6	6	39.	8204	15	2½	40.	7319	2	5¾
	232	1	11¼		211	6	8		1211	6	7½
	1067	17	9½		7743	13	3¼		3144	18	9½
	4032	12	1		6917	11	9½		2376	0	9½
	9416	9	8½		14376	15	10¼		163	9	11½
	1067	13	8		215	9	5¾		10782	17	1¾
	239	2	5¼		2935	8	1¼		6904	7	3½
	6409	12	8¼		9603	2	5½		109	19	10
	752	15	3½		6408	13	10½		4321	18	9½
	41263	9	5		32147	2	6		73041	2	7
	6197	11	7¾		9374	5	7		9276	3	8
	13257	8	11½		3925	4	11½		1528	3	6½
	3276	5	9¼		631	7	7¼		1397	7	1½
	46	3	6¾		691	13	9½		1140	15	6¾
	1287	14	7½		1414	12	4		10563	4	1½
	4917	10	8		5016	18	2		3198	10	8
	90365	10	4		436	2	3		78042	1	6
	360	5	5		56	4	10¾		25976	2	5
	1379	17	2¾		616	3	7½		6540	3	3¾
	9	9	10½		1189	2	3¾		7530	4	4½
	1340	16	9¼		401	3	6½		5811	5	5¾
	906	10	7½		7116	15	7¾		756	2	2½
	2222	5	1		917	18	8		76	11	10

The Civil Service Commissioners allow fifty marks for ' Separate Addition ' such as the above, with which the learner is requested to familiarise himself by repeated trials. It appears from carefully reading the Civil Service Reports, that persons are examined in ' Separate Addition ' who are candidates for Clerkships of every kind, Surveyorships of Taxes, Inspectorships of Factories, and Cadetships of Constabulary. Candidates for the situation of Letter-carriers and other subordinate offices, and also those for School Inspectorships, and for the Colonial Office, are not so tested.

	£	s.	d.		£	s.	d.		£	s.	d.
41.	431	7	9	42.	4028	11	3	43.	7603	3	7
	232	1	11¼		211	6	8		1211	6	7½
	1067	17	9½		7743	13	3¼		3144	18	9½
	4032	12	1		6917	11	9½		2376	0	9½
	9416	9	8½		14376	15	10¼		163	9	11½
	1067	13	8		215	9	5¾		10732	17	1¾
	239	2	5¼		2935	8	1¼		9673	1	0¼
	10397	8	7½		1528	3	6		109	19	10
	752	15	3½		6408	13	10½		4321	18	9½
	56735	12	8½		19058	15	1¾		25978	11	1½
	6197	11	7¾		9374	5	7		9276	3	8
	8416	9	10¼		7319	8	4¼		3617	6	8¼
	3276	5	9¼		631	7	7¼		1397	7	1½
	46	8	6¾		691	13	9½		1140	15	6¾
	1287	14	7½		1414	12	4		10563	4	1½
	4917	10	8		5016	18	2		3198	10	8
	90365	10	4		436	2	3		78042	1	6
	360	5	5		56	4	10¾		25976	2	5
	1379	17	2¾		616	3	7½		6549	3	3¾
	9	9	10½		1189	2	3¾		7530	4	4½
	1340	16	9¼		401	3	6½		5811	5	5¾
	906	10	7½		7116	15	7¾		756	2	2½
	2222	5	1		917	18	8		76	11	10

	£	s.	d.
4.	567	1	9
	232	1	11¼
	1067	17	9½
	4032	12	1
	9416	9	8½
	1067	13	8
	239	2	5¼
	6409	12	8¼
	752	15	3½
	41263	9	5
	4139	8	5
	13257	8	11½
	3276	5	9¼
	46	3	6¾
	1287	14	7½
	4917	10	8
	90365	10	4
	360	5	5
	1379	17	2¾
	9	9	10½
	1340	16	9¼
	906	10	7½
	2222	5	1

	£	s.	d.
45.	4032	11	3
	211	6	8
	7743	13	3¼
	6917	11	9½
	14376	10	10¼
	215	9	5¾
	2935	8	1¼
	9603	2	5½
	6408	13	10½
	32147	2	6
	8136	2	6
	3925	4	11½
	631	7	7¼
	691	13	9½
	1414	12	4
	5016	18	2
	436	2	3
	56	4	10¾
	616	3	7½
	1189	2	3¾
	401	3	6½
	7116	15	7¾
	917	18	8

	£	s.	d.
46.	8154	3	8
	1211	6	7½
	3144	18	9½
	2376	0	9½
	163	9	11½
	10732	17	1¾
	6904	7	3½
	109	19	10
	4321	18	9½
	73041	2	7
	3748	2	9
	1528	3	6½
	1397	7	1½
	1140	15	6¾
	10563	4	1½
	3198	10	8
	78042	1	6
	25976	2	5
	6549	3	3¾
	7530	4	4½
	5811	5	5¾
	756	2	2½
	76	11	10

	£	s.	d.
47.	632	5	11
	232	1	11¼
	1067	17	9½
	4032	12	1
	9416	9	8½
	1067	13	8
	239	2	5¼
	6409	12	8¼
	752	15	3½
	56735	12	8½
	6197	11	7¾
	40613	9	8
	3276	5	9¼
	46	8	6¾
	1287	14	7½
	4917	10	8
	90365	10	4
	360	5	5
	1379	17	2¾
	9	9	10½
	1340	16	9¼
	906	10	7½
	222	5	1

	£	s.	d.
48.	7125	13	4
	211	6	8
	7743	13	3¼
	6917	11	9½
	14376	15	10¼
	215	9	5¾
	2935	8	1¼
	9603	2	5½
	6408	13	10½
	19058	15	1¾
	9374	5	7
	7312	2	9
	631	7	7¼
	691	13	9½
	1414	12	4
	5016	18	2
	436	2	3
	56	4	10¾
	616	3	7½
	1189	2	3¾
	401	3	6½
	7116	15	7¾
	917	18	8

	£	s.	d.
49.	6147	1	8
	1211	6	7½
	3144	18	9½
	2376	0	9½
	163	9	11½
	10732	17	1¾
	6904	7	3¼
	109	19	10
	4321	18	9½
	25978	11	1½
	9276	3	8
	6341	7	10
	1397	7	1¼
	1140	15	6¾
	10563	4	1½
	3198	10	8
	78042	1	6
	25976	2	5
	6549	3	3¾
	7530	4	4½
	5811	5	5¾
	756	2	2½
	76	11	10

In almost all competitive examinations twelve columns, such as the preceding Nos. 38, 39, &c. are proposed to candidates.

COMPOUND SUBTRACTION.

48. Rule.—1°. Set down the less quantity under the greater, like two addends in the preceding rule.

2°. Commencing with the lowest denomination, take every term of the lower quantity from every term of the upper.

3°. If the lower term cannot be taken from the upper, borrow 1 from the next higher denomination of the upper line, then considering this so many *units* of the lower denomination, subtract, just as in *Simple Subtraction.*

Ex. 1. From £147 16*s.* 8½*d.* take £49 17*s.* 6¼*d.*

Here, we say, ¼ from ½, and ¼ remains, set down the ¼ under the farthings: again 6*d.* from 8*d.*, and 2*d.* remain, set down 2 under the pence. We cannot take 17*s.* from 16*s.*, therefore we borrow £1, calling it 20*s.*, this we add to 16*s.*, which makes 36*s.*, and say, 36—17 = 19*s.*, which we write under the shillings. Lastly, we carry 1 to the pounds and subtract as in Simple Subtraction.

£	*s.*	*d.*
147	16	8½
49	17	6¼
£97	19	2¼

Ex. 2. From 146 acrs. 2 r. 25 pls., take 19 acrs. 3 r. 30 pls.

Here, we take 30 from 40 + 25 = 65, as we must borrow one rood to enable us to subtract; or, we may say 30 from 40, and 10 remain; 10 and 25 are 35, which is set down. Carry 1 to 3 make 4, 4 from 3 we cannot, but 4 from 4 and 0 remains, 0 and 2 make 2, which is set down; carry 1 again, because we *borrowed* 1 acre, which we called 4 roods, and so on, as before.

acres	r.	pls.
146	2	25
19	3	30
126	2	35

X.

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
1.	109	7	4¼	2.	189	16	4¼	3.	1060	10	4½
	19	18	8½		86	17	5½		93	13	6¾

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
4.	2019	16	4¼	5.	107	10	4½	6.	123	13	11½
	198	14	8½		19	16	8¼		19	17	10¾

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
7.	2023	10	4¼	8.	3020	19	6¼	9.	1070	0	10
	127	12	9½		327	18	9½		57	18	0¾

	cwts.	qrs.	lbs.		cwts.	qrs.	lbs.		cwts.	qrs.	lbs.
10.	106	1	20	11.	194	1	16	12.	14	2	20
	19	2	18		14	2	19		9	3.	24

	acrs.	r.	pls.		acrs.	r.	pls.		acrs.	r.	pls.
13.	134	2	30	14.	96	3	4	15.	306	2	16
	97	2	36		19	2	30		119	2	25

	acrs.	r.	pls.		lbs.	oz.	drs.		lbs.	oz.	drs.
16.	407	1	10	17.	110	12	14	18.	79	10	9
	296	2	35		96	13	15		17	11	13

	lbs.	oz.	drs.		lbs.	oz.	drs.		yrs.	m.	wks.
19.	701	12	14	20.	90	13	8	21.	90	10	3
	19	12	15		17	10	11		19	11	2

	yrs.	m.	wks.		dys.	hrs.	m.		dys.	hrs.	m.
22.	112	9	2	23.	16	12	40	24.	90	20	30
	96	10	3		12	23	48		47	22	36

	oz.	dwts.	grs.		lbs.	oz.	dwts.		lbs.	oz.	dwts.
25.	119	16	20	26.	76	10	13	27.	17	3	4
	39	18	23		19	11	19		9	10	9

	oz.	dwts.	grs.		yds.	ft.	in.		mls.	f.	pls.
28.	11	14	20	29.	10	2	10	30.	16	6	30
	9	17	23		7	2	11		9	6	36

	sq.yds.	sq.ft.	sq.in.
31.	13	7	70
	10	8	100

	sq.pls.	sq.yds.	sq.ft.	sq.in.
32.	30	20	6	100
	10	25	6	120

	yds.	nls.	qrs.
33.	117	3	2
	19	3	3

	ells	qrs.	nls.	in.
34.	67	1	2	1
	19	2	3	1¼

Post Office (Junior Appointments).

	£	s.	d.		£	s.	d.		£	s.	d.
35.	103	5	4	36.	92	6	2¼	37.	72	3	6
	49	18	10		47	12	6¾		58	18	9

	£	s.	d.
38.	201	7	5½
	53	16	7¾

	£	s.	d.
39.	61	7	3
	58	18	9

	£	s.	d.
40.	301	8	3½
	53	16	7¾

Excise, Post Office, &c.

	£	s.	d.
41.	975	19	1¼
	9	16	10¾

		£	s.	d.
42.	From	453	6	2½
	Take	165	1	10¾

	£	s.	d.
43.	1687	1	7¾
	946	13	10½

	£	s.	d.
44.*	662303	11	3¼
	84606	16	4¾

Tidewaiter and Boatman.

	tons	cwts.	qrs.	lbs.
45.	359	17	2	19
	156	18	3	27

	yds.	ft.	in.
46.	252	1	1
	198	2	11

Examinations where 'Elementary Arithmetic' is required.

	£	s.	d.
47.	87341	10	9¼
	6442	19	11¾

	£	s.	d.
48.	641037	8	1¼
	83174	15	6¾

	£	s.	d.
49.	573156	11	2½
	92896	18	7¾

	£	s.	d.
50.	397612	4	5¼
	78360	16	9¾

	£	s	d.
51.	397612	4	5¼
	87406	18	6½

	mls.	fur.	pls.	yds.	ft.	in.
52.	38	7	13	1	1	2
	20	0	14	2	2	8

	mls.	fur.	pls.	yds.	ft.	in.
53.	79	6	12	1	1	6
	68	7	13	2	2	7

---

* Admiralty (Dockyards).

COMPOUND MULTIPLICATION.

Case I.

49. *When the multiplier does not exceed* 12.

Rule.—Set down the multiplier under the lowest denomination of the multiplicand. Multiply the number of this denomination and every other by it, setting down and carrying as in addition.

Ex. 1. Multiply £118 12*s.* 6¼*d.* by 4.

In this example we say 4 × 1 (read times or multiplied by) = 4*f.* = 1*d.*: set down nothing, but carry 1. Then, 4 × 6 = 24 and 1 = 25*d.* = 2*s.* 1*d.*: set down 1, and carry 2*s.* 4 × 12 = 48, and 48 + 2 = 50*s.* = £2 10*s.*: set down 10*s.* and carry £2 to the pounds, and so on.

£	*s.*	*d.*
118	12	6¼
		4
474	10	1

Ex. 2. Multiply 16 yds. 2 qrs. 3 nls. by 6.

Here, we say 6 × 3 = 18, and 18 nls. = 4 qrs. 2 nls.: we write down 2 and carry 4. Again, 6 × 2 = 12, and 12 + 4 = 16 qrs. = 4 yds.: set down 0 and carry 4.

yds.	qrs.	nls.
16	2	3
		6
100	0	2

XI.

Post Office.

	£	*s.*	*d.*		£	*s.*	*d.*
1.	35	3	7 × 5.	2.	28	15	4¾ × 7.
3.	54	7	9 × 5.	4.	92	16	7¾ × 6.
5.	67	5	11 × 5.	6.	84	13	9¾ × 6.

50. *To multiply by a composite number.*

Rule.—Multiply successively by its factors.

Ex. 3. £736 18*s.* 10½*d.* × 16.

Here, as 16 is = 4 × 4, we first multiply by 4, and then multiply this product by 4, which must be the product by 16.

£	*s.*	*d.*	
736	18	10½	
		4	
2947	15	6	= *prod. by* 4.
		4	
11791	2	0	= *prod. by* 16.

XI.

	£	s.	d.			£	s.	d.	
7.	709	19	7½	× 24 and by 27.	8.	590	9	11¾	× 48 and by 50.
9.	719	19	9¼	× 64 and by 72.	10.	718	16	9½	× 90 and by 96.
1.	427	17	7¾	× 120 and by 121.	12.	324	9	6½	× 132 and by 144.

CASE II.

51. *When the multiplier is greater than* 12, *but not a composite number.*

RULE.—Multiply by the nearest *composite* number, and add or subtract the product found by multiplying the multiplicand by the difference between this composite number and the given multiplier.

Ex. 4. Multiply £12 10*s.* 6½*d.* by 19.

Here, we find the product by 18, as in *Case I.*; then we add the multiplicand, that is, once itself, which must give the product by 19. It will be useful to prove the accuracy of this result, by taking the factors of 20 instead of 18.

£	s.	d.	
12	10	6½	× 19
		6	
75	3	3	= *prod. by* 6
		3	
225	9	9	= *prod. by* 18
12	10	6½	= *prod. by* 1
238	0	3½	= *prod. by* 19

Ex. 5. Multiply £16 13*s.* 14¼*d.* by 327.

In this example we first find the product by 100; this product is multiplied by 3, which gives the product by 300. To find the product by 20, we multiply the product by 10 by 2; lastly, we multiply the multiplicand by 7, and then add.

	£	s.	d.	
(7)	16	13	4¼	× 327
			10	
(2)	166	13	6½	= *prod. by* 10
			10	
	1666	15	5	= *prod. by* 100
			3	
	5000	6	3	= *prod. by* 300
	333	7	1	= *prod. by* 20
	116	13	5¾	= *prod. by* 7
	5450	6	9¾	= *prod. by* 327

	£	s.	d.
or,	16	13	4¼ × 327
	327		
	1962		
	327		
	5232		
10*s.* = £½	163	10	
2*s.* 6*d.* = ¼	40	17	6
10*d.* = ⅓	13	12	6
¼*d.* = 1/40	0	6	9¾
	£5450	6	9¾

This method presupposes a knowledge of division; otherwise it has many advantages.

Ex. 6. Multiply 7 ac. 2 r. 20 pls. by 497.

Here we multiply 20 by 10 = 200 pls. 5 rds., carry 5: looking at the 20, we see it is = one-half of a rood; we say, 10 half-rds. = 5 rds. and carry 5. In many examples we can proceed in a similar manner. We get the product by 90, by taking 9 times the product by 10.

	acs.	r.	pls.	
(7)	7	2	20	
			10	
(9)	76	1	0	= *prod. by* 10
			10	
	762	2	0	= *prod. by* 100
			4	
	3050	0	0	= *prod. by* 400
	686	1	0	= *prod. by* 90
	53	1	20	= *prod. by* 7
	3789	2	20	= *prod. by* 497

£ s. d.
13. 568 9 10¼ × 69, 74.
15. 360 19 7¼ × 137, 149.

£ s. d.
14. 336 17 10¼ × 95, 98.
16. 129 11 9¾ = 234, 378.

acrs. r. pls.
17. 19 3 30 × 147.

acrs. r. pls.
18. 23 1 16 × 523.

cwts. qrs. lbs.
19. 16 3 24 × 129.

cwts. qrs. lbs.
20. 16 1 27 × 729.

yds. qrs. nls.
21. 20 3 2 × 709.

yds. qrs. nls.
22. 27 3 3 × 999.

Tidewaiter and Weigher.

23. Multiply £227 10*s.* 5*d.* by 11.
24. Multiply 49 tons 13 cwts. 2 qrs. 26 lbs. 10 oz. 7 dwts. by 9.

Admiralty Dockyards.

Multiply £6807 16*s.* 4¾*d.* by 8.

Post Office, &c.

Multiply £20 3*s.* 0¾*d.* by 18.
Multiply £13 0*s.* 11½*d.* by 368.
Multiply £4693 11*s.* 10¾*d.* by 8.

Where 'Elementary Arithmetic' is required.

Multiply £6814 17*s.* 3½*d.* by 78.
Multiply £401 11*s.* 9½*d.* by 34, and by 43.
Multiply £6814 17*s.* 3½*d.* by 11.
Multiply £4016 11*s.* 2½*d.* by 94.
Multiply £6842 15*s.* 8½*d.* by 89.
Multiply £9043 5*s.* 11¼*d.* by 6, and by 7.

Education Office.

Multiply £1647 8*s.* 2¼*d.* by 635.

Various Departments.

Multiply £20954 17*s.* 6½*d.* by 465.
Multiply £1854 7*s.* 7½*d.* by 465.

COMPOUND DIVISION.

This rule must be viewed in two aspects.

To divide a compound quantity by an abstract num- which enables us to take the 3rd, 4th, or any part of ividend.

To divide one compound quantity by another of the kind, such as £1 5*s.* by 5*s.* This shows how often the end contains the divisor; and the quotient must s be an abstract number, expressing the number of the dividend contains the divisor. (*See note page 52.*) der the first head we have cases corresponding to those preceding rule.

CASE I.

53. *When the divisor is not greater than* 12.

RULE.—1°. Set down the divisor to the left of the dividend, as in Short Division.

2°. Divide the left-hand term of the dividend by the divisor, and set down the quotient under that term. If there is any remainder reduce it to the next lower denomination, adding to it, thus reduced, the number of that denomination, and divide the sum by the divisor, and so on with the other terms.

Ex. 1. Divide £14 12*s*. 6*d*. by 6.

We say 14 contains 6 twice, set down 2: the remainder £2 = 40*s*., and 40 + 12 = 52*s*.; now, 52 ÷ 6 = 8 + 4 r*, set down 8: these 4*s*. must be reduced to pence, then 4*s*. = 48*d*., and 48 + 6 = 54, 54 ÷ 6 = 9, therefore, the quotient is £2 8*s*. 9*d*.

	£	*s.*	*d.*	
6)	14	12	6	
	2	8	9	*Ans.*

XII.

Post Office.

1. £83 15*s*. ÷ 6.
2. £106 7*s*. 4½*d*. ÷ 9.
3. £88 9*s*. 6½*d*. ÷ 7.
4. £83 10*s*. 10½*d*. ÷ 6.
5. £81 3*s*. 1½*d*. ÷ by 7.
6. £77 11*s*. 4½*d*. ÷ by 6.

54. *When the divisor is a composite number.*

RULE.—Divide *successively* by its factors.

Ex. 2. £28 12*s*. 3*d*. ÷ 18.

Here we first divide by 6, and then by 3, both divisors coming under CASE I.

	£	*s.*	*d.*	
6)	28	12	3	
3)	4	15	4½	
	£1	11	9½	*Ans.*

7. £5698 17*s*. 6*d*. ÷ 40, and by 49.
8. £8066 9*s*. 6*d*. ÷ 72, and by 81.
9. £7899 15*s*. 0*d*. ÷ 360, and by 432.

CASE II.

55. *When the divisor is greater than* 12, *and not a composite number.*

---

* Remainder.

E.—Proceed as in *Case* I., using *Long Division* in-
f *Short*.

. Divide £167 16*s*. 1*d*. by 23.
eason of the annexed example is
We find 7, the first quotient
then, after reducing 6, the re-
, to shillings, we find 5, the next
figure. Again, after reducing
, we find 11, the last part of the
. The twenty-third part of
s. 1*d*. is £7 5*s*. 11*d*. This may
ed by multiplying the quotient by
sor.

```
      £   s.  d.
  23)167  16  1
     161
     ---  (£7 5s. 11d. Ans.
       6
      20
     ---
  23)136  (5
     115
     ---
      21
      12
     ---
  23)253  (11
     253
     ---
```

. Divide 109 cwts. 3 qrs. 13 lbs. by 46.
nly difference between
mple and the preceding
n the reduction; after
quotient figure we have
inder, which we bring
ers, adding the 3 qrs. in
iplicand, and so on with
r denominations. After
15 lbs., we have a re-
, 23, which we reduce
and find 8 oz.

```
     cwts. qrs. lbs.
  46)109   3   13
      92
      --  (2 cwts. 1 qr. 15 lbs. 8 oz. Ans.
      17
       4
      --
   46)71  (1 qr.
      46
      --
      25
      28
     ---
     203
      51
     ---
  46)713  (15 lbs.
     46
     ---
     253
     230
     ---
      23
      16
     ---
  46)368  (8 oz.
     368
     ---
```

XII.

£ s. d.

10. 794 16 5¼+ 78, and by 46.

11. 5760 15 4 +165, „ 196.

12. 9648 17 3¾÷427.

cwts. qrs. lbs.

13. 9006 3 20÷120.

14. 8693 2 26÷198.

15. 970 3 12÷563.

acrs. r. pls.

16. 1768 3 20+530.

17. 4960 2 35+625.

CASE III.

56. *To find how often one compound quantity contains another of the same kind.*[*]

RULE.—Reduce both to the same denomination, then divide as in Simple Division.

Ex. 5. Divide £1 11*s.* 6*d.* by 1*s.* 9*d.*

Here £1 11*s.* 6*d.*=378*d.*; and 1*s.* 9*d.*=21*d.*; then 378÷21=18. *Ans.*

Ex. 6. Divide 79 cwts. 2 qrs. 26 lbs. by 3 qrs. 10 lbs.

$$\left.\begin{array}{l}\text{Here, 3 qrs. 10 lbs.} = \text{94 lbs.}\\ \text{79 cwts. 2 qrs. 26 lbs.} = \text{8930 lbs.}\end{array}\right\}\text{then } 8930+94=95.\ \textit{Ans.}$$

£ *s.* *d.* £ *s.* *d.*

18. 1794 7 6÷3 12 6.

19. 2777 10 0+2 10 6.

£ *s.* *d.* £ *s.* *d.*

20. 2160 17 6÷1 9 6.

21. 3186 13 4÷2 13 4.

cwts. qrs. lbs. cwts. qrs. lbs.

22. 1468 3 7÷1 1 0.

cwts. qrs. lbs. cwts. qrs. lbs.

23. 1568 0 14+1 2 14.

CONTRACTIONS.

57. *When we have to divide by* 100, 1000, *&c.*

RULE.—Point off so many places to the *right* as there are *cyphers* in the divisor.

---

[*] Although one compound quantity cannot be multiplied by another, for, to take 9*d.* 10*d.* times, or to take £1 a shilling times, is absurd; yet we can find how often one compound quantity contains another of the same kind—we can find how often £5 contains 10*s.* by considering them abstract numbers when reduced to the same denomination.

At first sight it might appear that pounds are multiplied by 20*s.*, to reduce them to shillings; this is not the case; there will be 20 *times* as many shillings as pounds, therefore we multiply 'the number denoting how many pounds' by 20—not by 20*s.* more than by 20 crowns.

Ex. 7. £3818 17*s.* 1*d.* ÷ 100.

Because 3818 ÷ 100 = 38 + 18r, we can point off the 18 at once, then multiply this 18 by 20, and adding 17, we get 377; we point off 77, and multiply by 12, to reduce to pence, and then point off 25, which we multiply by 4, and point off 00.

£ *s.* *d.*
1,00)38,18 17 1
20
3,77 (£38 3*s.* 9¼*d.* *Ans.*
12
9,25
4
1,00

£ *s.* *d.*
24. 7179 13 6½ ÷ 100.

£ *s.* *d.*
25. 2327 16 8 ÷ 1000.

To divide by 800, 1200, 3600, *&c.*

Rule.—First divide by 8, &c., then divide by the other factor, 100.

Ex. 8. Divide £28150 10*s.* by 900.

Since 900 ÷ 9 = 100, we can first divide by 9, then by 100, as in the annexed operation.

£ *s.* *d.*
9)28150 10 0
1,00) 31,27 16 8
20
5,56
12
6,80
4
3,20, and 20 × 9 = 180*f.* = 3*s.* 9*d.*
£31 5*s.* 6¾*d.* + 3*s.* 9*d.* *Ans.*

£ *s.* *d.*
26. 14075 10 6 ÷ 800.

£ *s.* *d.*
27. 15816 15 0 ÷ 2400.

Post Office.

28. Divide £85 6*s.* by 72.

29. Divide £18 9*s.* 0¾*d.* by 78.

Education Office (London).

£ *s.* *d.*
30. 75919 11 7 ÷ 7.

£ *s.* *d.*
31. 611360 5 8 ÷ 623.

Admiralty.

£ *s.* *d.*
32. 648647 17 4 ÷ 32.

£ *s.* *d.*
33. 24727964 19 6 ÷ 238.

Various Departments in which 'Elementary Arithmetic' is to be known.

	£	s.	d.			£	s.	d.	
34.	13005	2	3	÷8 and by 7.	39.	8052	16	6	+44.
35.	8052	16	6	+22.	40.	1068091	12	11	+329.
36.	6842	15	8½	+89.	41.	1068091	12	11	+1316.
37.	2678492	15	6	÷18.	42.	40012	12	2¼	÷67.
38.	3496852	19	5½	÷94.					

DUODECIMALS.*

58. DUODECIMALS (Latin, *duodecim*, twelve) are so called because they increase by twelves from right to left. Although the various quantities in this rule increase in a twelve-fold ratio, yet the figures by which they are expressed are in the ordinary decimal notation, and are multiplied in the usual way.

In DUODECIMALS a foot is divided first into 12 equal parts called inches or primes, marked thus ($'$); and each of these again into 12 equal parts called seconds, marked thus ($''$). In like manner each second is divided into 12 equal parts called thirds ($'''$), each third into 12 equal parts called fourths ($''''$) and so on.

Hence $1'$ (inch or prime) $= \frac{1}{12}$ of a foot.
$1''$ (second), $\frac{1}{12}$ of $\frac{1}{12}$ $= \frac{1}{144}$ „
$1'''$ (third), $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ $= \frac{1}{1728}$ „
$1''''$ (fourth), $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ $= \frac{1}{20736}$ „

DUODECIMALS are added, subtracted, multiplied, and divided, just as other compound numbers, 12 of a less denomination making 1 of a greater, as in the following table:—

12 fourths ($''''$) = 1 third ($'''$)
12 thirds ($'''$) = 1 second ($''$)
12 seconds ($''$) = 1 inch or prime ($'$)
12 inches or primes ($'$) = 1 foot.

---

* This article cannot be thoroughly mastered until Fractions are known.

The marks $'$, $''$, $'''$, &c., are called *indices*, because they indicate or point out the denomination of the quantity to which they are attached.

59. Since feet stand in the place of units, feet multiplied by feet will give square feet as product: feet multiplied by inches or primes will give inches or primes, and so on, as follows:

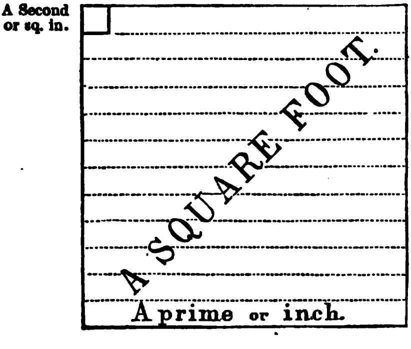
Feet multiplied by feet give feet as product.
Feet „ primes „ primes „
Feet „ seconds „ seconds „

Primes multiplied by primes give seconds.
Primes „ seconds „ thirds.
Primes „ thirds „ fourths.

Seconds multiplied by seconds give fourths.
Seconds „ thirds „ fifths.
Seconds „ fourths „ sixths.

In other words, the product will always be the denomination indicated by the *sum of the indices* of the multiplicand and multiplier: thus $7'' \times 5'''' = 35''''''$, thirty-five sixths.

The following geometrical figure will best illustrate what has just been said.



60. Duodecimals are principally used by workmen and artificers, in ascertaining the superficial and solid contents of their work.

The superficial content of a square or rectangular figure is found by multiplying the length by the breadth. The solid content or volume of a rectangular parallelopiped is found by multiplying the area of the end by the length.

Rule for Multiplying Duodecimals.

1°. Write the multiplier under the multiplicand, feet under feet, primes under primes, &c.

2°. Commencing with the highest denomination of the multiplier, multiply each denomination of the multiplicand by it separately, beginning at the lowest, setting down the products and carrying one for every twelve.

3°. Multiply by the next highest denomination in the multiplier in the same manner, setting down the products under the denominations of same kind, which will generally be effected by falling back one place to the right.

4°. The sum of the partial products of all the figures in multiplier, will be the total produced.

Ex. 1. In a log of timber 20 ft. 9 in. long, 2 ft. 5 in. broad, and 2 ft. 3 in. thick, how many cubic or solid feet are there?

ft.				
20	9′			
2	5′			
41	6′=product by 2ft.			
8	7′	9″=product by 5′		
50	1′	9″=product of length and breadth.		
2	3′			
100	3′	6″		
12	6′	5″	3‴	
112ft.	9′	11″	3‴	*Ans.*

In the above example we first say, 2ft. × 9′=18′=1 ft. 6′, we set down the 6′ and carry the 1 ft.: then twice 20 ft. + 1 = 41 ft., and so on.*

War Department.

Ex. 2. Multiply by the method of duodecimals 46 ft. 5 in. 6 parts by 9 ft. 7 in. 8 parts.

$$\begin{array}{rllll}
\text{ft.} & & & & \\
46 & 5' & 6'' & & \\
9 & 7' & 8'' & & \\
\hline
418 & 1' & 6'' = \text{product by 9 ft.} & & \\
27 & 1' & 2'' & 6''' = \text{product by } 7'. & \\
2 & 6' & 11'' & 8''' = \text{product by } 8''. & \\
\hline
447\text{ft.} & 9' & 8'' & 2''' \textit{ the answer.} &
\end{array}$$

Ex. 3. Express the result obtained in the last question in square feet, square inches, and a fraction of a square inch.

We have seen that $1' = 12''$ or square inches: therefore $9' = 108$ square inches; hence, we have, 447 ft. $9'\ 8''\ 2''' = 447$ ft. $108 + 8 + \frac{2}{12}$ sq. inches = 447 ft. $116\frac{1}{6}$ sq. inches. *Ans.*

War Department.

Ex. 4. The side of a solid cube is 7 ft. 3 in., find the content in solid yards, feet, and inches.

The content will be 7 ft. $3' \times 7$ ft. $3' \times 7$ ft. $3'$.

$$\begin{array}{rlll}
\text{ft.} & & & \\
7 & 3' & & \\
7 & 3' & & \\
\hline
50 & 9' & & \\
1 & 9' & 9'' & \\
\hline
52 & 6' & 9'' & \\
7 & 3' & & \\
\hline
367 & 11' & 3'' & \\
13 & 1' & 8'' & 3''' \\
\hline
381 & 0' & 11'' & 3'''
\end{array}$$

= 381 cubic ft. + 12 × 11 + 3 c. inches = 381 c. ft. 135 c. in: then, dividing by 27, we have, 14 c. yards 3 ft. 135 in. *Ans.*

---

* By fractions 20 ft. $9' \times 2$ ft. $5' = 20\frac{3}{4} \times 2\frac{5}{12} = \frac{83}{4} \times \frac{29}{12} = \frac{2407}{48} = 50\frac{7}{48}$ ft. = 50 ft. $1'\ 9''$ as before. If it were required to express the product 50 ft. $1'\ 9''$ in terms of a square foot, we would have $50 + (\frac{1}{12} + \frac{9}{144}) = 50 + \frac{12}{144} + \frac{9}{144} = 50\frac{21}{144} = 50\frac{7}{48}$ square feet, as above. Again, if we are to express the final product in cubic feet and fraction of a cubic foot, we have 112 cubic feet $+ \frac{9}{12} + \frac{11}{144} + \frac{3}{1728} = 112 + \frac{1296}{1728} + \frac{132}{1728} + \frac{3}{1728} = 112\frac{1431}{1728} = 112\frac{159}{192} = 112\frac{53}{64}$ cubic feet. It is also evident that the product could have been found by reducing both multiplicand and multiplier to inches; thus:—20 ft. $9' = 249$; 2 ft. $5' = 29$; then $249 \times 29 = 7221$, which ÷ 12 gives $601'\ 9''$, and this divided by 12 gives 50 ft. $1'\ 9''$.

XIII.

Proposed in Various Departments.

1. How many square yards, feet, and inches, are there in the top of a table the sides of which are 21 ft. 7½ in., and 19 ft. 5 in.?

2. How many square yards are there in a rectangular plot, the length of which is 36 ft. 9 in., and the breadth 24 ft. 10 in.?

3. The side of a cube is 17 in.; find its content in solid yards, feet, and inches.

4. How many square yards, &c., are there in a table 6 ft. 8 in. long, by 4 ft. 9 in. broad?

5. The area of a plot of ground is 101 yds. 3 ft. 90 in., and the length is 36 ft. 9 in.; find the breadth.

6.* The length of a French metre being 39·37 in., find the number of solid feet and inches in a cube the length of whose side is 2 metres.

7. Multiply by the method of duodecimals 7 ft. 5 in. 8 parts by 9 ft. 4 in. 11 parts.

8. Express the result obtained in the last question in square inches and a fraction of a square inch.

9. Multiply by duodecimals 2 ft. 7 in. 11 parts by 3 ft. 5 in. 7 parts, and explain clearly each term of the result.

10. The sides of a rectangular plot are 57 ft. 7 in. and 43 ft. 5 in.; find its area in square yards, feet, and inches.

MISCELLANEOUS EXERCISES IN THE COMPOUND RULES.

XIV.

1. How much does a collector receive in 2 years, his weekly collection being £7 4*s*. 3*d*.?

2. What will a labourer earn in 313 days, his wages being 11¾*d*. per day?

3. Divide the difference between a pound and a crown by 6.

4. From one thousand pounds take £982 14*s*. 0¼*d*., and multiply the remainder by 57.

5. Divide £100 between two persons, giving one £4 13*s*. 9*d*. more than the other.

6. Multiply £16 10*s*. 9*d*. by 4½ and divide the product by 13½.

---

* Proposed in Education Office.

7. Reduce 6 cwts. 5 sts. 7 lbs. to lbs. and divide the result by 14.

8. How long will £48 19*s.* 1½*d.* pay a labourer who earns 10½*d.* per day ?

9. How much was gained by selling a quantity of oats for £170 7*s.* 4¼*d.* which was purchased for £113 10*s.* 10½*d.* ?

10. What ground, at 4*s.* 8½*d.* per square perch, can be rented for £523 11*s.* 4*d.* ?

11. What quantity of tallow, at 7½*d.* per lb., could be purchased for £27 18*s.* 9*d.* ?

12. Bought 59 horses for £1040 19*s.* 7½*d.* ; what was the cost of each ?

13. How many dollars are there in £30 2*s.* 3*d.*, each dollar being worth 4*s.* 1½*d.* ?

14. From 347 tons 13 cwts. 2 qrs. 23 lbs. take 39 tons 15 cwts. 3 qrs. 25 lbs., and divide the remainder by 56.

15. What annual income would enable a person to expend 4*s.* 6*d.* per day, and save £2 6*s.* 10½*d.* per month ?

16. What was the salary of a person who paid £17 10*s.* income-tax when this tax was 14*d.* in the pound ?

17. How many shirts can be made from 60 ells of linen, each shirt requiring 3 yds. 3 qrs. ?

18. How much loaf sugar, at 8*d.* per lb., is equivalent in value to 30 sts. of moist sugar at 7*s.* 10*d.* per stone ?

19. How many dollars each equivalent to 4*s.* 2½*d.* should be received for £105 4*s.* 2*d.* ?

20. The price of a quantity of broadcloth at 10*s.* 8*d.* per yd. is £26 12*s.*, how much does it contain ?

21. What will be the cost of seeding 3 acres of potatoes at 6½*d.* per st., when 20 sts. are required for 30 perches ?

22. Divide £300 18*s.* 10½*d.* between two individuals, giving one £50 12*s.* 6*d.* more than the other.

Tidewaiters, &c.

23. Add £491 16*s.* 9*d.* ; £272 15*s.* 6¼*d.* ; £889 17*s.* 10½*d.* ; £647 19*s.* 2¾*d.* ; £398 16*s.* 7*d.* ; £563 16*s.* 10¼*d.* ; £770 0*s.* 5¾*d.* ; £945 17*s.* 7*d.* ; £240 13*s.* 9¼*d.* ; and £150 10*s.* together.

24. In 50 acres, how many sq. inches ?

25. In 3 leagues, how many inches ?

War Department.

26. Reduce £43 11*s.* 9¾*d.* to farthings.

27. Find the number of ounces in 18 tons 9 cwts. 3 qrs. 15 lbs.

28. If 1 yard of cloth cost 15*s.* 6¾*d.*, what will be the price of 32½ yds. at the same rate?

Post Office.

29. Find the value of 120 doz. of candles, at 9*s.* 9½*d.* per doz.

30. Find the price of 137 articles, at £1 17*s.* 6¼*d.* each.

31. In 42 mls. 4 fur. 3 pls. 4 yds., how many feet?

Excise.

32. Subtract 169 lbs. 6 oz. 14 dwts. 17 grs. from 187 lbs. 9 oz. 12 dwts. 20 grs.

33. Divide 467 tuns 3 hhds. 26 gals. 2 qrts. 1 pt. by 5.

Proposed to those who are to know 'Elementary Arithmetic,' except Letter-Carriers and Rural Messengers.

34. A sum of £10465 was divided between two persons, so that one had £547 10*s.* more than the other; what did each receive?

35. If a man rows at the rate of 7 miles an hour with the stream, the rate of which is 2½ miles, how fast will he row against it?

36. A person mixed 8 lbs. of tea at 3*s.* 6*d.*, 10 at 3*s.* 8*d.*, 12 at 4*s.* 2*d.*, and 10 at 4*s.* 6*d.*, and sold the mixture at 4*s.* 8*d.* per lb.; what did he gain out of the whole quantity?

37. Multiply 5 ft. 3 in. by 6 ft. 7 in., and the product by 7 ft. 11 in.

38. What is the actual value of the denomination which occupies the place of inches in this product?

Education and Constabulary Offices.

39. How often will a wheel, 3¼ yds. in circumference, turn between London and York, the distance being 198 miles?

40. How many solid yards, feet, and inches, are there in 175983 solid inches?

Police Courts, Prisons, &c.

41. Reduce 4 ac. 20 pls. to sq. feet.

42. How many miles, furlongs, &c., are there in 174082 inches?

43. How many lbs. oz., &c., are there in 228908 grs. of gold?

44. Reduce 4 miles 3 fur. 4 pls. 5 yds. to inches.

45. In 327471 oz., how many tons, cwts., &c.

Committee of Council on Education.

46. There are two mountains such that if 126 ft. are added to 4 times the height of the lower one, the sum is half the difference between their heights. Given that the lower one is 441 ft. high, find the height of the other.

47. What is the length of the longest day where the sun rises on that day at 3 hrs. 49 mins. 51 sec.?

Surveyorship of Taxes.

48. Reduce 14 tons 7 cwts. 1 qr. to ounces.

49. In 3784593 inches, how many miles?

Constabulary (Ireland).

50. In 6 tons 7 cwts. 3 qrs. 15 lbs., how many ounces?

51. If a man rows 10 miles in 2½ hrs. against the stream, the rate of which is 3 miles an hour; how long would he be in rowing 5 miles with the stream?

52. In 1721793 inches, how many miles, furlongs, &c.?

53. Find the circumference of a carriage wheel which revolves 1848 times in a journey of 3½ miles.

Where Candidates are to know 'Vulgar and Decimal Fractions.'

54. Reduce 6 miles 5 fur. 7 pls. 2 yds. to feet.

55. In 767914 sq. inches, how many yards, feet, and inches?

56. Reduce 36 miles 3 fur. 36 pls. 5 yds. to feet.

57. Reduce 63 cwts. 2 qrs. 5 oz. to drachms.

58. Reduce 14 lbs. 11 oz. 11 grs. to grains.

59. Reduce 787312 drachms to cwts., qrs., &c.

60. Reduce 13 yds. 5 ft. 19 in. (cubic measure) to inches.

Various Departments.

61. How many feet are there in 9 miles 3 fur. 6 pls. 1 yd.?

62. How many lbs. of silver are there in 270 spoons, each of which weighs 1 oz. 13 dwts. 8 grs.?

63. In 1847638 inches, how many miles, &c.?

64. How many francs, at 9½*d.* each, are there in £693 14*s.* 9*d.*?

65. Reduce 46 lbs. 3 oz. 2 dwts. 2 grs. to grains.

66. Reduce 17 tons 13 cwts. 1 lb. to ounces avoirdupois.

67. In 537086 inches, how many miles, furlongs, &c.?

Admiralty.

68. In 36845371 oz. avoirdupois, how many tons, cwts., &c.?

Tidewaiters and Weighers.

69. In 259566 lbs. avoirdupois weight, how many tons, cwts., and qrs.?

70. In 1749134 seconds, how many weeks, days, &c.?

VULGAR FRACTIONS.

61. A FRACTION is an expression for a part or parts of anything considered as a whole.

A *fraction* may be either greater or less than a *unit*; if it be *one-half* of *ten* it is equal to *five units.*

A *fraction* is expressed by means of two numbers placed one over the other with a line between them.

Thus, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{8}{5}$, read, *one-half, two-thirds, eight-fifths.*

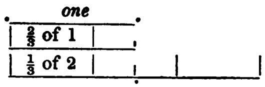
The number below the line is called the *Denominator,* because it shows the number of parts into which the whole or unit is divided. It gives the name or denomination to the fraction, just as *cwt.* gives the name *hundredweight* to any number after which it is written. The number above the line is called the *Numerator.* The numerator shows or enumerates the number of parts expressed by the fraction. If we divide anything into four equal parts, we express three of these parts by the fraction $\frac{3}{4}$. The numerator and denominator are called the *Terms* of the fraction.

62. A fraction corresponds to an example in division before the process is performed, the *numerator* corresponding to the *dividend* and the *denominator* to the *divisor.* Therefore the TRUE OR REAL VALUE of any fraction is the *quotient* obtained by dividing its numerator by its denominator. Hence, the greater the numerator in comparison with the denominator, the greater the value of the fraction. For, the more parts we take of anything, the greater must be the particular fraction used to express these parts.

Increasing the numerator, the denominator remaining the same, increases the value of the fraction: it increases the number of parts; or, which amounts to the same thing, it increases the dividend, the divisor remaining the same. Increasing the denominator decreases the value of the fraction; as it makes a greater number of parts, each part must be smaller. It increases the divisor, the dividend remaining the same.

The *terms* of a fraction may be both multiplied or both divided by the same number, without affecting the value of the fraction. The quotient will remain the same, therefore the value of the fraction will not be changed. For, $\frac{6}{3}=\frac{24}{12}=\frac{48}{24}$, &c.

Two-thirds of one is equal to *one-third* of two, as may be understood from the illustration.



For, $\frac{2}{3}$ of a shilling $=8d.$, and $8d.$ = one-third of $24d.$, or 2 shillings.

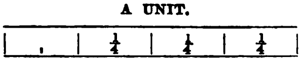
In the same manner it may be shown that $\frac{1}{2}$ of $1=\frac{1}{4}$ of 2, and $\frac{3}{4}$ of $1=\frac{1}{4}$ of 3, &c.

63. A PROPER FRACTION has its numerator less than its denominator:—$\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, are proper fractions.

An IMPROPER FRACTION has its numerator equal to or greater than its denominator, and is equal to or greater than a unit:—$\frac{4}{4}$, $\frac{5}{4}$, $\frac{7}{3}$, are improper fractions.

A SIMPLE FRACTION represents a part or parts of one or more units; as we have seen, $\frac{2}{3}$ may be considered $\frac{2}{3}$ of 1, or $\frac{1}{3}$ of 2.

A COMPOUND FRACTION represents a part or parts of a fraction, and is known by having 'of' interposed; as, $\frac{1}{2}$ of $\frac{1}{4}$, $\frac{1}{3}$ of $\frac{1}{2}$, &c. It bears a similar relation to a fraction that a simple fraction bears to a whole number.



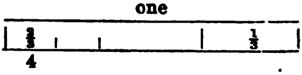
Observe that the $\frac{1}{4}$ to the left is divided into two parts, each of which represents the fraction $\frac{1}{2}$ of $\frac{1}{4}$. Again, since $\frac{1}{4}$ of a shilling $=3d.$, and one-half of $3d.=1\frac{1}{2}d.$, and $1\frac{1}{2}d.=\frac{1}{8}$ of a shilling, therefore $\frac{1}{2}$ of $\frac{1}{4}=\frac{1}{8}$.

A MIXED NUMBER consists of a whole number and a fraction; as, $3\frac{1}{2}$, $2\frac{3}{4}$, $6\frac{1}{3}$.

A **COMPLEX FRACTION** represents a part or parts of a fraction, of a mixed number, or a fractional part of a whole number; as,

$$\frac{\frac{2}{3}}{4} \quad \frac{4}{\frac{2}{3}} \quad \frac{3}{4\frac{1}{2}} \quad \frac{\frac{1}{2}\text{ of }3}{2+\frac{1}{3}}$$

The first means that one-fourth of $\frac{2}{3}$ is to be taken, and not $\frac{1}{4}$ of 1. The second expresses $\frac{2}{3}$ of 4, and not $\frac{2}{3}$ of 1. They will be easily understood from the following explanation.



Since $\frac{2}{3}$ of a shilling $= 8d.$, $\frac{\frac{2}{3}}{4} = 8d. \div 4 = 2d. = \frac{1}{6}$ of a shilling, therefore $\frac{\frac{2}{3}}{4} = \frac{1}{6}$. Also, $\frac{2}{3}$ of a pound $= 13s.\ 4d.$, and $13s.\ 4d. \div 4 = 3s.\ 4d.$ $= \frac{1}{6}$ as before.

REDUCTION.

64. *To reduce an improper fraction to its equivalent whole or mixed number.*

RULE.—Divide the numerator by the denominator, the quotient is the whole number; under the remainder (if any) write the denominator, in the form of a fraction.

Ex. 1. Reduce $\frac{15}{4}$ and $\frac{79}{14}$ to whole numbers.

Here, $15 \div 4 = 3 + 3$ r. $= 3\frac{3}{4}$ *Ans.*

And, $14)79\ (5\frac{9}{14}$ *Ans.*

$$\begin{array}{r} 70 \\ \hline 9 \\ \hline \end{array}$$

REASON.—The denominator shows the number of parts to be taken to make up a unit; therefore, as many parts must be put together to make 1, as there are units in the denominator; this is effected by the rule. Since the operation of division does not alter the nature of the quantity divided, the remainder is a part of the given fraction, and is expressed as such by writing the denominator under it.

XV.

Reduce to whole or mixed numbers:—

1. $\frac{23}{5}$, $\frac{29}{7}$, $\frac{33}{17}$.
2. $\frac{1362}{25}$, $\frac{7025}{35}$, $\frac{801}{11}$.
3. $\frac{2045}{144}$, $\frac{1969}{19}$, $\frac{2347}{21}$.
4. $\frac{19}{2}$, $\frac{365}{17}$, $\frac{649}{47}$.

65. *To reduce a mixed number to its equivalent improper fraction.*

Rule.—Multiply the whole number by the denominator of the fraction, and to the product add its numerator; under this sum write the denominator of the fraction.

Ex. 1. Reduce $14\frac{11}{30}$ to an improper fraction.

$$\begin{array}{r} 14\frac{11}{30} \\ 30 \\ \hline 431, \end{array} \text{ then } \frac{431}{30} \textit{ Ans.}$$

This rule is the reverse of the preceding. The *reason* is the same in principle as that for the reverse operation.

Reduce to improper fractions:—

5. $7\frac{1}{9}$, $7\frac{3}{5}$, $18\frac{3}{7}$.
6. $129\frac{11}{40}$, $16\frac{19}{23}$, $5\frac{7}{9}$.
7. $18\frac{10}{17}$, $70\frac{1}{5}$, $69\frac{5}{11}$.
8. $32\frac{1}{9}$, $16\frac{7}{12}$, $113\frac{117}{119}$.

66. Any number may be written in a fractional form by putting 1 under it for a denominator.

We may give a whole number *any* denominator, if we first multiply it by that denominator.

For, $8=8\times\frac{6}{6}=\frac{48}{6}$, and $10=\frac{50}{5}$; also, $20=20\times\frac{5}{5}=\frac{100}{5}$.

9. Reduce 19, 10, 30, to fractions whose denominators shall be 11, 17, 29, respectively.

10. Reduce 27, 39, 165, to fractions whose denominators shall be 39, 46, 57, respectively.

We may also reduce a fraction to another fraction having *any* denominator, if the denominator of such a fraction be a *factor* of the number to which it is to be reduced.

Thus, $\frac{1}{4}$ can be brought to a fraction whose denominator is 16; since $16=4\times4$, we have merely to multiply the terms of the fraction by 4, and we have $\frac{1\times4}{4\times4}=\frac{4}{16}$.

We cannot, however, reduce $\frac{1}{4}$ to a simple fraction whose denominator shall be 18, since 4 is not one of the factors of 18.

We may reduce $\frac{1}{2}$ and $\frac{7}{8}$ to equivalent fractions, having 48 for denominator. For, $\frac{1}{2}=\frac{24}{48}$ (multiplying the terms by 24), and $\frac{7}{8}=\frac{42}{48}$ (multiplying the terms by 6).

67. If one number divides another without a remainder, it is said to be a MEASURE of that other.

A *Measure* is another name for *Factor.*

If any number be a measure or factor of two or more others, it is called a *common measure* of them; that is, a measure common to them. The numbers 12 and 18 have each the following measures:—12 has 12, 6, 4, 3, 2; 18 has 18, 9, 6, 3, 2. Therefore 12 and 18 have 6, 3, 2, common measures. 6 is their *greatest common measure,* that is, the greatest measure common to them.

68. *To find the greatest common measure of any two numbers.*

RULE.—1°. Divide the greater number by the less; divide the less by the remainder; continue the process, always dividing the preceding divisor by the remainder, until there is no remainder.

2°. The last divisor will be the greatest common measure.*

Ex. 1. Find the greatest common measure of 12 and 18.

We divide by 12, and get 6 for remainder; we then divide 12, the first divisor, by 6, and find no remainder: therefore 6 is the greatest common measure of 12 and 18.

12)18(1
12
6)12(2
12

---

* Before we prove the reason of the rule it is necessary to show:—

I. 'If a number measure two other numbers it will measure their sum and difference.' Thus, 4 is a measure of 12 and of 20, it is a measure of 32 (12+20), and also a measure of 8 (20—12).

II.—'If one number *measure* another, it will measure any multiple of that other.' (A multiple of a number is its product when multiplied by any number.) Thus, 4 is a measure of 12; it is also a measure of 24 (12×2), and of 36 (12×3).

III. The *greatest common measure* that two or more numbers can have is the least of those numbers.

To find the G. C. M. of 72 and 156. The G. C. M. these two numbers can have is 72 (III.); hence, if 72 measures 156, it is their G. C. M. We find by trial that 72 does not measure 156; therefore their G. C. M. is less than 72, otherwise it would not be a common measure. In trying if 72 measures 156, that is, in dividing by 72, we have divided 156 into

72)156(2
144
12(72(6
72

XV.

Find the greatest common measure of:—

11. 66 and 154; and of 195 and 819.
12. 378 and 1467; and of 18996 and 29984.
13. 365 and 7345; and of 1964 and 78688.

For more than two numbers.

Rule.—Find for *two*, then of this measure and a *third*, and so on.

Find the greatest common measure of:—

14. 12, 42, and 66; and of 19, 57, and 380.
15. 324, 612, and 1032; and of 96, 216, 88, and 16.
16. 14, 70, and 1484; and of 8128, 14816, 75288, and 8472.

69. *To reduce fractions to their lowest terms.*

Rule I.—Divide the terms of the fraction by any common factors; the new terms in the same manner, and so on, until terms are obtained which have no common factors.

Rule II.—Divide the terms of the fraction by their greatest common measure.

Ex. 1. Reduce $\frac{48}{112}$ to its lowest terms.

Here, $\frac{48}{112}=\frac{24}{56}=\frac{12}{28}=\frac{6}{14}=\frac{3}{7}$ *Ans.* This result is found by dividing by 2 continually. But we might have divided by 4, or by any other common factor.

The greatest common measure of 48 and 112, we find is 16; then dividing the terms by 16, we get $\frac{3}{7}$, as before.

The following observations will be found useful in finding common factors:—

1. All even numbers are divisible by 2 at least.
2. All even numbers are divisible by 4, when the number expressed by their last two figures is divisible by 4.

---

two parts; one is a multiple of 72, the other (12) is the difference between that multiple and 156. Whatever measures 144 and 12 will measure 156 (I.); but the G. C. M. of 144 and 12 may not measure 72; therefore, instead of trying 144 and 12 we try 72 and 12, as we know the number that measures 72 will measure 144 (II.). Now, the *greatest* measure that 72 and 12 can have is 12 (III.), which we find will answer. It is obvious that 12 is the G. C. M. of 72 and 156, since the greatest possible measure was tried in every case.

In dividing the greater number by the less, that *multiple* of any divisor may be taken which comes nearest the dividend, whether greater or less than it. We may divide any divisor and its corresponding dividend by common factors, at any step during the process.

3. A number is divisible by 8, if its *units' period* be divisible by 8.

4. An equal number of cyphers cancel when numbers end in one or more cyphers.

5. Numbers ending in 0 or 5, are divisible by 5, and no others.

6. If the *sum* of all the digits of a number be divisible by 3 or 9, the number is divisible by 3 or 9.

7. A number is divisible by 11, if the *sums* of the alternate digits be equal, beginning at either right or left; or, if one sum exceed the other by 11, or 22, 33, &c.*

XV.

Reduce to their lowest terms:—

17. $\frac{72}{168}$, $\frac{48}{180}$, $\frac{272}{1496}$. 19. $\frac{5544}{8008}$, $\frac{48016}{108036}$, $\frac{25025}{35035}$.

18. $\frac{1638}{24102}$, $\frac{1572}{1965}$, $\frac{7956}{9724}$. 20. $\frac{49}{294}$, $\frac{1235}{2223}$, $\frac{13068}{15444}$.

Proposed in various Departments.

21. Reduce $\frac{7040}{7392}$, to its lowest terms.

22. „ $\frac{6398}{7312}$, to its lowest terms.

23. „ $\frac{69615}{92872}$, to its lowest terms.

THE LEAST COMMON MULTIPLE.

70. When a number is a *factor* of another number, it has been called a *measure* of that other.

A number is called a MULTIPLE of any of its measures or factors. Thus 12 is a *multiple* of 6, and of 4.

---

* REASON.—1. This is self-evident.

2. Since 100 is divisible by 4, any number of hundreds is divisible by 4. But any number may be considered as a certain number of hundreds + the number expressed by the figures in its units' and tens' places. Therefore if these last be divisible by 4, the entire number must be divisible by 4. Thus, 1728=1700+28; 736=700+36.

3. The reason of this is similar to that of the preceding, only say 1000 instead of 100, and 8 instead of 4.

4. This is obvious.

5. Go over the MULTIPLICATION TABLE by 5.

6. Every number is equal to two other numbers, one of which is a multiple of 9 and the other the sum of its digits; thus, 324=3×100+2×10+4, and since 10=9+1, 100=99+1 &c., we have, 324=3(99+1)+2(9+1)+4=297+18+3+2+4=315 (a multiple of 9)+9 (the sum of the digits).

When a number is a multiple of two or more others, as 12 of 6, 4, and 3, it is called their *Common Multiple.*

The numbers 6, 4, and 3, have also 24, 36, 48, &c., common multiples. For simplicity we always endeavour to use the *least* common multiple of numbers.

71. *To find the Least Common Multiple of two or more numbers.*

RULE.—1°. Place the numbers in a line; then divide by a measure or divisor, found by inspection, which will divide the largest number of the given numbers.

2°. Set down the quotients and undivided numbers.

3°. Proceed in the same manner with the quotients and undivided numbers, until no number can be found to measure any two of them.

4°. Multiply together the undivided numbers and the divisors used; the product will be the least common multiple.

When numbers are all *prime*, that is, when they have no common factors, their product is their least common multiple.*

Ex. 1. Find the least common multiple of 6, 12, 9, 18, 24, and 30.

$$\begin{array}{rrrrrrr}
3) & 6 & 12 & 9 & 18 & 24 & 30 \\ \hline
2) & 2 & 4 & 3 & 6 & 8 & 10 \\ \hline
2) & 1 & 2 & 3 & 3 & 4 & 5 \\ \hline
3) & & 1 & 3 & 3 & 2 & 5 \\ \hline
 & & & 1 & 1 & 2 & 5
\end{array}$$

By looking at the numbers, we see at once 3 will measure more of them than any other common factor; we divide by 3, and write the quotients under the dividends. We next divide by 2; and these quotients again by 2; these again by 3; until we find 2 and 5—numbers which have no common factors. The product of quotients is, $3 \times 2 \times 2 \times 3 = 36$; and this $36 \times 2 \times 5$, the undivided numbers, gives 360, the required least common multiple.

$$\overset{\textit{divisors.}}{3 \times 2 \times 2 \times 3}, \overset{\textit{undivided numbers.}}{\times 2 \times 5} = 360.$$

---

* When finding the *least common multiple* of a set of numbers, any of the numbers may be *rejected* that are measures of one of the given numbers. Thus in Ex. 1, the numbers 6, 12, and 9, may be *rejected*, and the least common multiple of 18, 24, and 30 found. In Ex. 2, *reject* 2, 4, and 8, and find for 16 and 18. When no number will measure the denominators of a set of fractions, it is evident that the *least common multiple* will be found by multiplying all the denominators together.

Reason.—This process is merely using *one* of the factors common to several numbers instead of *all* the factors the same as itself: therefore, as only *prime* and *common* factors are used, the product is the *least possible*; and since no factors have been *omitted*, the product must be a *multiple of all* the numbers.

72. *To reduce fractions to equivalent ones having a Common Denominator.*

Rule.—Find the least common multiple of all the denominators, which will be the required common denominator; then proceed as in **66.**

This is merely an *extension* of **66**, and we proceed on the same principle.

Reason.—We have seen that all the denominators must measure that denominator to which they can be reduced: hence the reason of the rule.

Ex. 1. Reduce $\frac{1}{2}$ and $\frac{1}{3}$ to equivalent fractions having the same denominator.

Here, $\frac{1}{2}=\frac{3}{6}$, and $\frac{1}{3}=\frac{2}{6}$; therefore $\frac{3}{6}$ and $\frac{2}{6}$ *Ans.*

In this operation we multiply the terms of each fraction by the same number; therefore we do not alter the value of any of the fractions used. The multiplier for the terms of any fraction will always be the *quotient* obtained by dividing the least common multiple by the denominator of that fraction. In the fractions $\frac{1}{2}$ and $\frac{1}{3}$, 6 is the least common multiple of 2 and 3; then $6\div2=3$, the multiplier for $\frac{1}{2}$; and $6+3=2$, the multiplier for $\frac{1}{3}$. Then $\frac{1\times3}{2\times3}=\frac{3}{6}$, and $\frac{1\times2}{3\times2}=\frac{2}{6}$, as before.

Ex. 2. Reduce $\frac{1}{2}$, $\frac{1}{4}$, $\frac{5}{8}$, $\frac{11}{16}$, $\frac{5}{18}$, to others having a common denominator.

We write the denominators in a line, and find their least common multiple 144, which is the denominator of their equivalent fractions. We find the multipliers thus:—$144\div2$, the denominator of the first, $=72$, the numerator of the equivalent to $\frac{1}{2}$; again, $144\div4=36$. For the third, we say $144\div8=18$, and $18\times5=90$, the numerator of the equivalent to $\frac{5}{8}$; and so on.

$$\frac{1}{2}=\frac{72}{144}\quad \frac{1}{4}=\frac{36}{144}\quad \frac{5}{8}=\frac{90}{144}\quad \frac{11}{16}=\frac{99}{144}\quad \frac{5}{18}=\frac{40}{144}$$

$$\begin{array}{r|rrrrr} 4) & 2 & 4 & 8 & 16 & 18 \\ \hline 2) & 2 & 1 & 2 & 4 & 18 \\ \hline & 1 & 1 & 1 & 2 & 9 \end{array}$$

Then $4\times2\times2\times9=144$, the *least common multiple.*

Reduce the following to their *equivalents* having the same denominator: *—

XV.

24. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$.
25. $\frac{3}{4}$, $\frac{7}{12}$, $\frac{5}{16}$, $\frac{7}{8}$.
26. $\frac{3}{11}$, $\frac{17}{22}$, $\frac{4}{5}$, $\frac{19}{33}$.
27. $\frac{7}{9}$, $\frac{8}{11}$, $\frac{13}{15}$, $\frac{9}{17}$.
28. $\frac{4}{7}$, $\frac{5}{9}$, $\frac{11}{18}$, $\frac{13}{15}$.

73. ***To express one quantity as the fraction of another of the same kind.***

RULE.—1°. Write the former as numerator, the latter as denominator.

2°. If they are of different denominations, they must be reduced to the same denomination, and for simplification the fraction reduced to its lowest terms.

Ex. 1. What fraction of 18*s.* is 6*s.*? Here $\frac{6}{18}=\frac{1}{3}$ *Ans.*

Ex. 2. What fraction of 2*s.* is 2¼*d.*?

Here, $\frac{2\frac{1}{4}d.}{2s.}=\frac{2\frac{1}{4}d.}{24d.}=\frac{9f.}{96f.}=\frac{3}{32}$ *Ans.* or $\frac{9}{4}$ × [illegible]

Reduce to the fraction of a *pound*:—

29. 17*s.* 6*d.*; 15*s.* 9*d.*; 1*s.* 10½*d.*
30. 2*s.* 9*d.*; 1*s.* 7½*d.*; 9*s.* 3*d.*

Reduce to the fraction of a *shilling*:—

31. 9*d.*, 7½*d.*, 4½*d.*

Reduce to the fraction of a *cwt.*:—

32. 2 qrs. 14 lbs.; 3 qrs. 14 lbs.; 1 qr. 7 lbs.

Reduce to the fraction of an *acre*:—

33. 2 r. 20 p.; 3 r. 10 p.; 1 r. 30 p.; 3 r. 25 p.

34. Express 2*s.* 0¾*d.* as a fraction of 4*s.* 9¾*d.*
35. Express £2 6*s.* 8*d.* as a fraction of £3 13*s.* 4*d.*
36. What fraction of £27 17*s.* 1¼*d.* is £23 11*s.* 4¾*d.*?

---

* It is necessary that we should be able to bring fractions to the *same* denomination or kind without altering their value; for fractions, like any other quantities, can neither be added nor subtracted when they are of different kinds. Making them have a common denominator is reducing them to others *equal in value*, but all of the same kind. By looking at $\frac{5}{7}$ and $\frac{7}{9}$ we cannot say which is the greater, but since $\frac{5}{7}=\frac{45}{63}$, and $\frac{7}{9}=\frac{49}{63}$, we see at once that $\frac{7}{9}$ is the greater.

ADDITION.

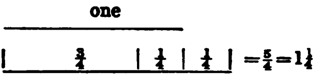
CASE I.

74. ***When the fractions have a common denominator.***

RULE.—Add the numerators together and under their sum write the common denominator.

Ex. 1. Add $\frac{3}{4}$ and $\frac{2}{4}$. Here we say $2+3=5$, and write 4 the common denominator under it; $\frac{5}{4}=1\frac{1}{4}$ *Ans.*

REASON.—Since $\frac{3}{4}s.=9d.$, and $\frac{2}{4}s.=6d.$, we have $\frac{3}{4}+\frac{2}{4}=9d.+6d.=15d.=1\frac{1}{4}s.$ Or, because $\frac{3}{4}=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$, and $\frac{2}{4}=\frac{1}{4}+\frac{1}{4}$; we have $\frac{3}{4}+\frac{2}{4}=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{5}{4}=1\frac{1}{4}$; compare this with the illustration. The fractions to be added should always be written as in **72**, *Ex.* 2.



XVI.

Add the following:—

1. $\frac{1}{4}+\frac{3}{4}+\frac{1}{4}$; $\frac{3}{7}+\frac{6}{7}+\frac{5}{7}+\frac{4}{7}$.
2. $\frac{7}{11}+\frac{6}{11}+\frac{9}{11}$; $\frac{11}{13}+\frac{12}{13}+\frac{6}{13}$.

CASE II.

75. ***When the fractions to be added have not a common denominator.***

RULE—1°. Reduce them to others equal in value, and having a common denominator.

2°. Add these others as in CASE I.

Customs and·Excise.

Ex. 1. Add $\frac{3}{4}$, $\frac{5}{8}$, $\frac{2}{3}$, together.

As we cannot add 4ths to 8ths, nor to 6ths, we must find a *common denominator* 24 and reduce the fractions into equivalent ones having a common denominator, namely to 24ths. Adding these we get $\frac{49}{24}=2\frac{1}{24}$ *Ans.*

$$\begin{array}{r|rrr} 4 & 4 & 8 & 3 \\ \hline & 1 & 2 & 3 \end{array}$$

$4\times2\times3=24$ *common denominator.*

$\frac{3}{4}=\frac{18}{24}$
$\frac{5}{8}=\frac{15}{24}$
$\frac{2}{3}=\frac{16}{24}$

$\frac{49}{24}=2\frac{1}{24}$

Various Departments.

Ex. 2. Add together $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{12}$.

The explanation is similar to that given in Ex. 1, and the work will be as the annexed.

$$\begin{array}{r|rrrr} 3) & 3 & 4 & 5 & 12 \\ \hline & 1 & \cancel{4} & 5 & 4 \end{array}$$

$3 \times 5 \times 4 = 60$ *common denominator.*

$$\begin{array}{l} \quad\underline{60}\text{ C. D.} \\ \frac{2}{3} = 40 \\ \frac{3}{4} = 45 \\ \frac{4}{5} = 48 \\ \underline{\frac{7}{12} = 35} \\ \quad\frac{168}{60} = 2\frac{4}{5}.\ \textit{Ans.} \end{array}$$

In actual practice the denominators of the equivalent fractions are not usually written.

Various Departments.

3. Add together $\frac{1}{4}$, $\frac{5}{12}$, $\frac{7}{15}$, $\frac{3}{20}$.

Post Office.

4. Add the following $\frac{3}{4}$, $\frac{2}{7}$, and $\frac{5}{8}$.

Excise.

5. Add $\frac{3}{4}$, $\frac{2}{9}$, and $\frac{5}{8}$ together.
6. „ $\frac{3}{4}$ and $\frac{6}{7}$ together.

War Office.

7. Add together $\frac{7}{16}$, $\frac{7}{15}$, and $\frac{11}{13}$.

Customs and Excise.

8. Add $\frac{5}{9}$, $\frac{3}{4}$, and $\frac{2}{3}$ together.
9. „ $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{9}$ together.

Audit Office.

10. Add $\frac{2}{3}$ and $\frac{5}{7}$ together.

Inland Revenue.

11. Add together $\frac{9}{10}$, $\frac{5}{7}$, and $\frac{4}{5}$.

76. *To add mixed numbers.*

Rule.—Add the fractional parts by the rules given above, set down the fraction of the sum, if any, and carry the *whole number*, if any, to the sum of the whole numbers.

India Board.

Ex. 3. Add together $7\frac{1}{3}$, $\frac{3}{14}$, $\frac{10}{21}$, and $2\frac{7}{9}$.

In this example the fractions when added make $1\frac{101}{126}$; we set down the fraction $\frac{101}{126}$ and carry the 1 to the whole numbers, and find the answer $10\frac{101}{126}$.

$$\begin{array}{r|rrrr} 7) & 3 & 14 & 21 & 9 \\ \hline 3) & 3 & 2 & 3 & 9 \\ \hline & 1 & 2 & 1 & 3 \end{array}$$

$7 \times 3 \times 2 \times 3 = 126$ *common denominator.*

$$\begin{array}{rcr} & & 126 \text{ C. D.} \\ 7\frac{1}{3} & = & 42 \\ \frac{3}{14} & = & 27 \\ \frac{10}{21} & = & 60 \\ 2\frac{7}{9} & = & 98 \\ \hline 10\frac{101}{126} & & \frac{227}{126} = 1\frac{101}{126} \end{array}$$

Audit Office.

12. Add $\frac{1}{5}$, $4\frac{1}{3}$, and $\frac{2}{5}$ together.

Customs.

13. Add together $1\frac{3}{8}$, $\frac{1}{6}$, $\frac{5}{18}$, and $2\frac{1}{12}$.

Various Departments.

14. Add together $\frac{3}{4}$, $\frac{5}{7}$, $1\frac{5}{6}$, $2\frac{1}{8}$.
15. „ $\frac{4}{5}$, $\frac{5}{6}$, $4\frac{5}{9}$, $1\frac{7}{10}$.
16. „ $\frac{2}{5}$, $\frac{3}{7}$, $\frac{4}{11}$, and $\frac{2}{13}$ of $7\frac{4}{5}$.
17 „ $3\frac{1}{9}$, $5\frac{3}{5}$, $1\frac{4}{15}$, $\frac{2}{3}$.

Science and Art, Prisons, Post Office, &c.

18. Add together $5\frac{4}{9}$, $\frac{3}{8}$, $\frac{7}{12}$, and $2\frac{5}{24}$.

Various.

19. Add together $7\frac{1}{8}$, $\frac{2}{3}$, $3\frac{4}{5}$, $1\frac{11}{12}$.

Admiralty.

20. Add together $\frac{4}{5}$, $\frac{7}{8}$, $\frac{11}{12}$, $3\frac{3}{20}$.

Various Examinations.

21. Add together $\frac{2}{5}$, $2\frac{5}{6}$, $2\frac{1}{8}$, $\frac{5}{12}$.
22. „ $3\frac{5}{12}$, $3\frac{1}{6}$.

Constabulary.

23. Add together $9\frac{1}{7}$, $1\frac{3}{4}$, $1\frac{2}{21}$, and $2\frac{13}{42}$.

SUBTRACTION.

CASE I.

77. *When the fractions have a common denominator.*

RULE.—Take the numerator of the less fraction from the numerator of the greater, and under the remainder write the common denominator.

Ex. 1. From $\frac{5}{6}$ take $\frac{1}{6}$.

Take 1 from 5 and 4 remain, under which we write 6; then $\frac{4}{6}=\frac{2}{3}$ *Ans.*

$$\begin{array}{l} \frac{5}{6} \\ \frac{1}{6} \\ \hline \frac{4}{6}=\frac{2}{3}\ \textit{Ans.} \end{array}$$

REASON.—$\frac{1}{6}s.=2d.$, $\frac{5}{6}=10d.$; hence $\frac{5}{6}-\frac{1}{6}=10d.-2d.=8d.=\frac{8}{12}=\frac{2}{3}$ as before.

XVII.

1. From $\frac{5}{8}$ take $\frac{3}{8}$; also $\frac{11}{12}-\frac{7}{12}$; $\frac{4}{7}-\frac{2}{7}$.
2. „ $\frac{13}{14}$ „ $\frac{11}{14}$; „ $\frac{8}{19}-\frac{5}{19}$; $\frac{13}{16}-\frac{7}{16}$.

CASE II.

78. *When the fractions have not a common denominator.*

RULE.—Bring the fractions to equivalent ones having a common denominator, then proceed as in CASE I.

Commissariat.

Ex. 2. Subtract $\frac{5}{12}$ from $\frac{7}{8}$.

In this question we find 24 the common denominator, and the given fractions are (**72**) equivalent to $\frac{21}{24}$ and $\frac{10}{24}$; then $21-10=11$, under which we write the common denominator. Or, since $\frac{1}{8}$ of a shilling $=1\frac{1}{2}d.$, $\frac{7}{8}=10\frac{1}{2}d.$; and since $\frac{1}{12}$ $s.=1d.$, $\frac{5}{12}=5d.$ then $10\frac{1}{2}d.-5d.=5\frac{1}{2}d.=\frac{11}{24}s.$ as before.

$$\begin{array}{l} \frac{7}{8}=\frac{21}{24} \\ \frac{5}{12}=\frac{10}{24} \\ \hline \quad\ \frac{11}{24}\ \textit{Ans.} \end{array}$$

Inland Revenue.

3. Subtract $\frac{3}{10}$ from $\frac{8}{11}$.

Customs.

4. Subtract $\frac{7}{8}$ from $\frac{9}{10}$.

79. *To subtract mixed numbers.*

Rule.—1°. Subtract just as in pence and farthings.

2°. Take care to *borrow* one when the fraction of the smaller number is greater than the other fraction: this 1 always must be considered as divided into the number of units expressed by the common denominator.

Proposed where Fractions are to be known.

Ex. 3. Subtract $5\frac{7}{10}$ from $8\frac{1}{2}$.

As we cannot take $\frac{7}{10}$ from $\frac{5}{10}$ we borrow 1 from the 8 and call it $\frac{10}{10}$ its equivalent. We then say $\frac{7}{10}$ from $\frac{15}{10}$ $(\frac{10}{10}+\frac{5}{10})$ and $\frac{8}{10}$ remain $=\frac{4}{5}$. We proceed in a similar manner in every example having a fraction in the subtrahend greater than that in the minuend. If there be no fraction in the minuend we subtract the numerator of the fraction in subtrahend from the common denominator.

$$\begin{array}{lr} \text{From } 8\frac{1}{2} = 8\frac{5}{10} & \\ \text{Take } \underline{5\frac{7}{10} = 5\frac{7}{10}} & \\ & 2\frac{8}{10} = 2\frac{4}{5}\ \textit{Ans.} \end{array}$$

Various Departments.

5. Subtract $5\frac{3}{7}$ from $7\frac{2}{3}$.
6. Subtract $11\frac{7}{18}$ from $16\frac{2}{15}$.

Customs.

7. Subtract $\frac{3}{4}$ from $1\frac{4}{25}$.

Excise.

8. Subtract $4\frac{1}{3}$ from $6\frac{8}{15}$

India Board.

9. Subtract $5\frac{6}{35}$ from $7\frac{4}{5}$.

Post Office, Prisons, &c.

10. Subtract $1\frac{17}{20}$ from $4\frac{3}{7}$.

Various Departments.

11. Subtract $8\frac{3}{7}$ from $18\frac{2}{3}$.
12. Subtract $6\frac{5}{12}$ from $9\frac{3}{4}$.

Admiralty.

13. Subtract $19\frac{7}{8}$ from $32\frac{3}{4}$.
14. Subtract $1\frac{7}{9}$ from $5\frac{3}{4}$.

Constabulary.

15. Subtract $7\frac{7}{45}$ from $11\frac{3}{20}$.

Surveyorship of Taxes.

16. Subtract $3\frac{3}{8}$ from $5\frac{11}{12}$.
17. Subtract $6\frac{9}{10}$ from $27\frac{19}{20}$.

MULTIPLICATION.

80. *To multiply fractions.*

RULE.—Multiply the numerators together for the numerator of the product, and the denominators for its denominator.

Customs.

Ex. 1. Multiply $\frac{3}{4}$ by $\frac{7}{8}$.

Here, $\frac{3}{4}\times\frac{7}{8}=\frac{3\times7}{4\times8}=\frac{21}{32}$ *Ans.*

Compound fractions are reduced to *Simple* fractions in the same manner.

Thus $\frac{2}{3}$ of $\frac{5}{6}=\frac{2}{3}\times\frac{5}{6}=\frac{10}{18}=\frac{5}{9}$; and $\frac{1}{2}$ of $\frac{4}{5}=\frac{1}{2}\times\frac{4}{5}=\frac{4}{10}=\frac{2}{5}$.

The operation may be contracted by dividing any numerator and any denominator by common factors. Thus $\frac{15}{16}\times\frac{8}{15}=\frac{1}{2}\times\frac{1}{1}=\frac{1}{2}$, found by dividing the numerator of the first and the denominator of the second by 15, and denominator of first and numerator of second by 8.

This rule includes the multiplication of whole numbers, for $2\times3=\frac{2}{1}\times\frac{3}{1}=\frac{6}{1}=6$. It also includes the multiplication of a whole number by a fraction, or a fraction by a whole number; in either case, we write the whole number in a fractional form and multiply. Thus, $\frac{5}{6}\times3=\frac{5}{6}\times\frac{3}{1}=\frac{15}{6}=2\frac{3}{6}=2\frac{1}{2}$, and $3\times\frac{5}{6}=\frac{3}{1}\times\frac{5}{6}$, as before $=2\frac{1}{2}$.

It follows from the method of contraction explained above that when the whole number by which we are to multiply is a *measure* of the *denominator* of the fraction, we have merely to divide that denominator by the whole number for the product; thus,

$$\frac{5}{6}\times3=\frac{5}{6\div3}=\frac{5}{2}=2\frac{1}{2}.$$

81. In the same manner the value of a fraction of any quantity can be found, by considering that quantity as numerator of a fraction, and then multiplying the quantity by the numerator of the given fraction, and dividing the product by its denominator.

Thus, to find $\frac{4}{5}$ of £3; we say £$\frac{3}{1} \times \frac{4}{5} =$ £$\frac{12}{5} =$ £2 8*s.*

By first principles, $\frac{4}{5}$ of £3; here $\frac{1}{5}$ of £3 = £3 ÷ 5 = 12*s.*: therefore $\frac{4}{5} =$ 12*s.* × 4 = £2 8*s.*

REASON.—Take the example $\frac{1}{2} \times \frac{3}{4}$. Now, $\frac{1}{2} \times 3 = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$ by *addition*; but we have seen that $\frac{3}{4}$ is $= \frac{1}{4}$ of 3; therefore we have multiplied by a quantity *four times* too great, and hence our product $\frac{3}{2}$ is *four times* too great. Now, $\frac{3}{2}$ is the quotient of 3 by 2; this quotient we still require to divide by 4; but any number divided by 2 and then by 4, is the same as dividing by 8; therefore $\frac{3}{8}$ is the answer, which is the rule.

Mixed numbers should be reduced to improper fractions, and whole numbers written in fractional forms, before multiplying.

XVIII.

Inland Revenue.

1. Multiply $\frac{3}{10}$ by $\frac{8}{11}$.

Customs and Excise.

2. Multiply $\frac{3}{8}$ by $\frac{3}{9}$.

Commissariat.

3. Multiply $\frac{5}{9}$ of $\frac{8}{12}$ by $\frac{6}{7}$.

Post Office, Science and Art, Prisons, &c.

4. Multiply $2\frac{3}{4}$ by $5\frac{20}{21}$.

Customs and Excise.

5. Multiply $\frac{1}{4}$ of $\frac{3}{8}$ by $\frac{5}{7}$.
6. „ $\frac{3}{4}$ of 9 by $\frac{7}{8}$.

Expectants of Excise.

7. Multiply $\frac{2}{7}$ by $\frac{17}{45}$.

India Board.

8. Multiply $22\frac{2}{11}$ by $4\frac{2}{5}$.

Admiralty.

9. Multiply $161\frac{1}{8}$ by $2\frac{3}{4}$.

Constabulary.

10. Multiply $10\frac{2}{11}$ by $8\frac{1}{4}$.

Surveyor of Taxes.

11. Multiply $18\frac{2}{9}$ by $5\frac{7}{18}$.
12. „ $13\frac{2}{7}$ by $19\frac{1}{12}$.

Various Departments.

13. Multiply $9\frac{3}{7}$ by $3\frac{2}{11}$.
14. „ $10\frac{3}{7}$ by $3\frac{11}{12}$.
15. „ $11\frac{2}{5}$ by $8\frac{3}{7}$
16. „ $11\frac{5}{9}$ by $7\frac{7}{8}$.
17. „ $42\frac{3}{5}$ by $\frac{10}{27}$.
18. „ $9\frac{4}{5}$ by $5\frac{5}{7}$.
19. What is the value of $\frac{5}{36}$ of a guinea?
20. Find the value of $\frac{2}{3}$ of £1 + $\frac{3}{7}$ of £2 10*s.* 2*d.* + $2\frac{1}{3}$ of 16*s.*
21. Add together $\frac{4}{15}$ of a pound, $3\frac{3}{13}$ of a shilling, and $\frac{3}{13}$ of a penny.
22. Multiply $19\frac{5}{6}$ by $3\frac{3}{17}$.

DIVISION.

82. *To divide one fraction by another.*

RULE.—Invert the divisor and multiply.

Thus, $\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times \frac{5}{3} = \frac{10}{9} = 1\frac{1}{9}$: $\frac{7}{12} \div \frac{5}{8} = \frac{7}{12} \times \frac{8}{5} = \frac{56}{60} = \frac{14}{15}$.

Mixed numbers must be reduced to improper fractions and compound fractions to simple ones, before applying the rule. A *Complex* fraction is a short mode of expressing the division of one fractional quantity by another, a fractional quantity by a whole number, or a whole number by a fractional quantity. *Complex* fractions are reduced to *Simple* ones by this rule.

Thus, $\dfrac{\frac{2}{3}}{\frac{4}{5}} = \frac{2}{3} + \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$.

Ex. 2. Divide $\frac{9}{10}$ by 3.

Here, $\frac{9}{10} \div 3 = \frac{9}{10} \div \frac{3}{1} = \frac{9}{10} \times \frac{1}{3} = \frac{9}{30} = \frac{3}{10}$ *Ans.*

From the above example the following Rules will be easily derived.

83. *To divide a fraction by a whole number or the contrary.*

Rule.—Either multiply its denominator or divide its numerator by that number.

84. *To divide any quantity by a fraction.*

Rule.—Multiply the quantity by the denominator of the fraction, and divide the product by the numerator.

For, $3 \div \frac{4}{5} = \frac{3}{1} \times \frac{5}{4} = \frac{15}{4} = 3\frac{3}{4}$; also, 3*s.* 6*d.* $\div \frac{4}{5} = \frac{3s.\ 6d.}{1} \times \frac{5}{4} = \frac{17s.\ 6d.}{4} =$ 4*s.* 4½*d.*

Reason.—It is evident that fractions having a common denominator may be divided like any other quantities, by omitting the denominators, and dividing the numerators only. Thus, $\frac{2}{3} \div \frac{3}{5} = \frac{10}{15} \div \frac{9}{15} = 10 \div 9 = \frac{10}{9} = 1\frac{1}{9}$; but $\frac{10}{9} = \frac{2}{3} \times \frac{5}{3}$, which is the rule.*

XIX.

Find the quotient of:—

1. $\frac{2}{9} \div \frac{7}{8}$, and of $\frac{1}{4} \div \frac{1}{3}$.
2. $1\frac{1}{9} \div 2\frac{5}{6}$, and of $10 \div \frac{8}{11}$.
3. $16\frac{3}{4} \div 9$, and of $\frac{19}{28} \div \frac{4}{9}$.

---

* If we divide $\frac{2}{3}$ by 3; the fraction $\frac{2}{3}$ implies the quotient of 2 by 3, which quotient is still to be divided by 3; but dividing twice by 3 is the same as dividing by 9; hence $\frac{2}{3} \div 3 = \frac{2}{9}$. Now if the divisor be $\frac{3}{5}$ instead of 3, it is evident that we have divided by a quantity 5 times too great; consequently, the quotient is 5 times too small; hence we must multiply $\frac{2}{9}$ by 5, $= \frac{10}{9}$; then $\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times \frac{5}{3} = \frac{10}{9}$, which is the rule.

When one proper fraction is *divided* by another, the quotient is greater than the dividend. Division, like Multiplication of Fractions, implies *multiplication* by one number, and *division* by another; and in proper fractions the multiplier is greater than the divisor.

In dividing a mixed number by a whole number, proceed as in the annexed example. After the whole number, 8, is found, the remainder is $3\frac{3}{4}$; then, *mentally*, say, $3\frac{3}{4} = \frac{15}{4}$, write 15 for numerator, and say, 8×4=32, denominator.

$$\begin{array}{l} 8)\underline{67\frac{3}{4}} \\ \quad 8\frac{15}{32}\ \textit{Ans.} \end{array}$$

We should be prepared to avail ourselves of such examples as $24\frac{1}{2} \div 7 = 3\frac{1}{2}$. Here, $24\frac{1}{2}$ contains 7, 3 times, with $3\frac{1}{2}$ remainder; then, since $3\frac{1}{2} = \frac{1}{2}$ of 7, we can write $\frac{1}{2}$ at once. Again, $62\frac{1}{2} \div 10 = 6\frac{1}{4}$; as the remainder, $2\frac{1}{2}$, is $\frac{1}{4}$ of 10, we write $\frac{1}{4}$ at once.

When the divisor is a fraction, the quotient will express '*what part or parts the dividend is of the divisor.*'

Find the quotient of:

4. £7 12*s.* 6*d.* ÷ $\frac{11}{12}$.
5. £16 13*s.* 4*d.* ÷ $\frac{7}{9}$.
6. 16 cwts. 2 qrs. 14 lbs. ÷ $\frac{5}{7}$.
7. 4 acrs. 2 rds. 15 pls. ÷ $\frac{6}{7}$.

Reduce to simple fractions:—

8. $\dfrac{\frac{3}{4}+\frac{1}{2}}{\frac{3}{4}}$; $\dfrac{\frac{3}{5}}{4} \div \frac{4}{7}$; $\dfrac{\frac{1}{2}+\frac{1}{4}}{\frac{1}{3}\times\frac{1}{4}} \div \frac{2}{3}$.

9. $\dfrac{\frac{7}{8}}{\frac{4}{5}} \div 7$; $\dfrac{4\frac{1}{2}}{3\frac{1}{4}} \div \dfrac{\frac{1}{2}+\frac{1}{4}}{\frac{2}{3}}$, $\frac{7}{9} \div \dfrac{\frac{1}{2} \text{ of } \frac{1}{3}}{7}$.

85. *To reduce a fraction of one quantity to the fraction of another of the same kind.*

RULE.—If from a lower denomination to a higher, divide: if from a higher to a lower, multiply the fraction by as many of the lower as make one of the higher denomination.

Ex. 1. Reduce $\frac{2}{3}$*d.* to the fraction of a shilling.

Here, $\frac{2}{3} \div 12$, the number of pence in a shilling, $=\frac{2}{36}=\frac{1}{18}$. *Ans.*

Ex. 2. Reduce $\frac{2}{3}$ of a crown to a fraction of a shilling.

Here, $\frac{2}{3} \times 5 = \frac{10}{3} = 3\frac{1}{3}$*s.* *Ans.*

10. Reduce £$\frac{7}{8}$ to the fraction of £2 10*s.*
11. Reduce $\frac{3}{4}$*s.* to the fraction of 10*d.*
12. Bring $\frac{4}{7}$ lb. to the fraction of 2 qrs.

Commissariat.

13. Divide $\frac{6}{7}$ by $\frac{2}{9}$.

Customs.

14. Divide $\frac{4}{3}$ by $\frac{5}{6}$.

Expectants of Excise.

15. Divide $\frac{3}{4}$ by $\frac{8}{10}$.
16. „ $\frac{6}{20}$ by $\frac{4}{5}$.
17. „ $\frac{41}{33}$ by $\frac{63}{11}$.
18. „ $\frac{9}{16}$ by $4\frac{1}{2}$.

Admiralty.

19. Divide $15\frac{5}{12}$ by $7\frac{5}{9}$.

Surveyor of Taxes.

20. Divide $4\frac{1}{3}$ by $7\frac{11}{13}$.
21. „ $7\frac{5}{9}$ by $4\frac{2}{19}$.

Various Departments.

22. Divide $2\frac{4}{15}$ by $4\frac{1}{3}$.
23. „ $4\frac{5}{6}$ by $6\frac{7}{18}$.
24. „ $7\frac{5}{7}$ by $4\frac{2}{21}$.
25. „ $18\frac{2}{7}$ by $5\frac{1}{7}$.
26. „ $5\frac{1}{6}$ by $3\frac{2}{3}$.
27. What fraction of half-a-crown is $\frac{2}{5}$ of 6*s*. 8*d*.
28. Reduce £7 9*s*. 6*d*. to the fraction of £13 4*s*. 6*d*.
29. Reduce 6 acres 1 rood and 4 poles to the fraction of $2\frac{1}{2}$ roods.
30. What fraction of a pound is 9*s*. 10*d*. ?

Customs, Post Office, &c.

31. Add $\frac{3}{4}$ of a pound to $\frac{5}{6}$ of a shilling.
32. From $\frac{1}{8}$ of a yard take $\frac{2}{3}$ of an inch.

DECIMALS.

86. We now come to consider another class of fractions, called DECIMAL FRACTIONS, or more briefly DECIMALS. Any fraction whose denominator is 10 or any power of 10, or, more simply, which has 1 followed by any number of cyphers for its denominator, is called a DECIMAL FRACTION.

The denominators of decimals are generally *suppressed*; hence it becomes necessary to have a method of distinguishing their numerators from whole numbers. If we omit the denominator of $\frac{7}{10}$ it becomes 7; to prevent this 7 from being read 7 units, we place a point before it; thus, ·7.

87. A more extended *Numeration Table* must now be given, but almost *identical* with that of whole numbers (**9.**)

THOUSANDS.	UNITS.		THOUSANDTHS.	MILLIONTHS.
0 0 0	0 0 0	•	0 0 0	0 0 0
••• •• •	••• •• •		••• •• •	••• •• •

This Table, like that just referred to, has an *indefinite* number of periods; each period has *three* places—units, tens, hundreds. Therefore decimals may be *written* and *read* like whole numbers. This Numeration Table also increases towards the left. In fact it is merely a continuation of the former table to the right; and, like it, increases in a tenfold ratio to the left. In the above table, the period immediately to the left of the *units' period* is the *thousands'*, and that immediately to the right the *thousandths';* because every 1 in this period is the one-thousandth part of a unit; just as 1 unit is the one-thousandth part of 1000, and 1000 the one-thousandth part of a million. In the same manner 12 in this period are twelve-thousandths, and so on. The next period to the right is called the *millionths'* period; we may continue to form periods to the right as far as we please.

If it is required to write 165 thousandths, we must place it in its *proper* period; for this purpose prefix the *point*; thus, ·165, read, *one hundred and sixty-five thousandths.* To set down 3 thousandths, prefix 00, to have 3, that is, 3 *units of thousandths* in its proper place; thus, ·003. We read 3·63, three units and six hundred and thirty thousandths.

88. If the point be removed *one* place to the *right*, the number will be increased *tenfold.* For 6 = ten times ·6, and ·6 = ten times ·06. This is evident from the nature of notation. On the contrary, if the point be removed *one* place to the *left*, the number will be divided by 10; if two places, by 100; and so on. When the point is removed one place to the right the units become tens, &c.

Any number of cyphers to the right of a decimal has no effect, as they do not alter the position of the point. Thus, ·6 = ·60 = ·600. But a cypher placed between the point and a decimal diminishes the decimal tenfold; for ·6 = ten times ·06: two cyphers, placed in this position, decrease the decimal one hundred-fold: for ·6=one hundred times ·006.

With a little practice the learner will be able to write decimals with the same facility as whole numbers.

XX.

Write down in figures :—

1. Ninety-five thousandths.
2. Thirty-nine thousandths.
3. Twelve units and sixty-eight thousandths.
4. Six hundred and nine thousandths.
5. Fifty units and sixty thousandths.
6. Twenty-nine units and nine thousandths.
7. Forty-seven units and four hundred thousandths.
8. Sixty-nine units six hundred and seven thousandths and sixty-four millionths.
9. One hundred and twenty units fourteen thousandths and nineteen millionths.
10. Fifty-five units six hundred and forty-nine thousandths and four hundred millionths.
11. Sixty units sixty thousandths and sixty millionths.
12. Seventeen units seventeen thousandths and seventeen millionths.

ADDITION.

89. RULE.—1°. Set down the addends with their points in the same vertical line.

2°. Then add as in *Simple Addition*, and place the point of the sum under the other points.

REASON.— It has been shown that the *notation* of decimals is exactly the same as that of whole numbers; therefore we may say, '*Add and Subtract as in whole Numbers.*'

Various Departments.

Ex. 1. Add together 3·406, ·0212, 47·9 and ·6.

This is added just as an example in whole numbers; care should be taken in writing the decimal points directly under each other.

```
 3·406
  ·0212
47·9
  ·6
───────
51·9272 Ans.
```

XXI.

Various Departments.

1. Add together 52·38, 367·4, ·172, and 6·0053.
2. „ 672·5, 4·923, 80, and ·076.

Customs.

3. Add together 2·8146, ·0938, 8, ·875, 31·2788, 4·0087.
4. „ 1·3046, 15·031, ·0082, and 3·29.

Various Departments.

5. Add 74·36, 5·473, ·9872, 340·03, and 5984.
6. Add 72·5, 2·1574, 371·4, and 2·75.

India Board.

7. Add together 42·79, ·2105, ·047, and 140.

Various Departments.

8. Add together 27·03, 452·0091, ·37, 1·873592, and 83.
9. „ 70·1046, 701, ·6, ·16, and 7·304.
10. „ 501·1306, ·96, 6·401302, and 72.
11. „ 35·2176, 201·00541, 3·1482, ·054, 216·945, 7543·4.

Science and Art, Prisons, &c.

12. Add together 62·7, 2·30073, 450·923.

Constabulary.

13. Add together ·813, 420·91, ·0093, 7·043, 12560.

SUBTRACTION.

90. RULE.—Write the numbers as two addends are written: subtract as in whole numbers: and place the *point* in the remainder under the other points.

Various Departments.

Ex. 1. Subtract 19·7564 from 26.

In this example we may suppose cyphers annexed to the minuend, as such cyphers have no effect (**88**) on its value. In subtracting we begin just as in whole numbers and place the point in its proper place as directed by the rule.

26·0000
19·7564
6·2436 *Ans.*

XXII.

India Board.

1. Subtract 42·946 from 161·06.

Expectants of Excise.

2. From 10154271· take 9178965·8742.
3. From 991742865·010101 take 881745522·201002.

Inland Revenue.

4. Subtract 17·2463 from 29.
5. „ 337·55 from 350·111.

Constabulary.

6. From 506·222 take 59·87.

Science and Art, War Department, Prisons.

7. Subtract 5·74 from 121·0239.

Admiralty Dockyards.

8. Subtract 3·901 from 6·01.

Various Departments.

9. From 3017·215 take 5·7124.
10. Subtract 83·450392 from 1210·3.
11. Subtract 901·53629 from 30640·48.
12. Add together 5·3, ·00015, 643·8797, 62·09, 1035; and subtract 109·000365 from the result.
13. Subtract 236·932 from 270·00086.

MULTIPLICATION.

91. RULE.—1°. Multiply as in whole numbers disregarding the points.

2°. Point off so many places of decimals in the product as are in *both* multiplicand and multiplier, prefixing cyphers if necessary.

Ex. 1. 7·64 × 8·9.

In this example there are two places of decimals in multiplicand and one in multiplier; therefore the product must have three places of decimals.

```
  7·64
   8·9
 -----
  6876
 6112
 -----
67·996 product.
```

2. Multiply ·476 by ·0137.

his example after multiplying we prefix 00 to up 7 places of decimals, the number in both licand and multiplier.

```
     ·476
     ·0137
     ─────
     3332
    1428
   476
  ───────
  ·0065212 product
```

SON.—The nature of this rule, as well as its reason, will be best stood from the following explanation.

ve have to multiply 7·9 by 6, that is, to add 7·9, 6 times, we say =5·4 : this can be shown by *Addition*. Set down ·4, and carry 5 ; ore 7·9 × 6 = 47·4, and has only one decimal place. Now, if we the multiplier ten times less, we have 7·9 × ·6, which must give a ct ten times less than 47·4, that is 4·74 having two places of als. Also, 7·9 × ·06 = ·474 ; and 7·9 × ·006 = ·0474, having four al places, which establishes the rule.

XXIII.

Customs and Excise.

1. Multiply 75·6 by 6·75.
2. „ 19·378 by 19·38.
3. „ 33·21 by 4·41.
4. „ 5·874 by 58·139.
5. „ 1·021 by ·0037.

Post Office, Police Courts, &c.

6. Multiply 3·46 by ·489.
7. „ 79·004 by ·00473.

India Board.

8. Multiply 65·43 by ·00376.

Admiralty Dockyards.

9. Multiply 6·4073 by ·42.

Constabulary Office.

10. Multiply 757·04 by 15·8.

Various Departments.

11. Multiply 9·436 by 67·49.
12. „ 4·81 by ·0074.

13. Multiply 3·05 by ·25 and ·32 by ·231.
14. „ 1·342 by ·2057.
15. „ 5412·384 by 1·0023.
16. „ ·1356 by ·1458.
17. „ 5·0103 by 6·503.
18. „ 80·46 by ·00392.

DIVISION.

92. RULE.—I.—1°. If the dividend and divisor do not contain an equal number of decimal places, make the number of places equal by annexing cyphers.

2°. Then disregarding the points, divide as in whole numbers; the quotient thus obtained will be a whole number; after which the point is placed.

3°. Any other quotient figures, found by adding cyphers, will be decimals.

Ex. 1. Divide 478·9875 by 7·5.

There are four places of decimals in the dividend, we annex 000 to the divisor to have an equal number in it. Having brought down 5, the last figure of the dividend, and having found its corresponding quotient figure, we write the point, and add cyphers for the remaining figures of the quotient.

```
7·5000)478·9875
       450000
       ------(63·865
        289875
        225000
        ------
         648750
         600000
         ------
          487500
          450000
          ------
           375000
           375000
           ------
```

93. *When the dividend contains more places of decimals than the divisor.*

RULE.—1°. Set a mark after an equal number of decimal places in the dividend.

2°. Then divide as in *Rule I.*, remembering to bring down the other figures of the dividend in the process.

Ex. 2. 478·9875÷7·5.

This is the same example as given under *Rule I.* Compare the two methods and the superiority of this one when applicable will be evident.

7·5)478·9,875(63·865 *Ans*
450
289
225
648
600
487
450
375
375

XXIV.

Customs and Excise.

1. Divide 3·24 by ·25.
2. „ 2·5 by ·32.
3. „ 9065 by ·049.

Science and Art, War Department, &c.

4. Divide 307·8 by 23·8 to 3 places of decimals.
5. „ 8·747 by 413·4 to 3 places of decimals.

Post Office and Excise, &c.

6. Divide 476·532 tons by 60 and bring it to its proper value.*
7. „ £880·7643 by 5·48 and bring it to its proper value.

India.

8. Divide 154·28 by ·0064.

Admiralty.

9. Divide 240·13 by 73·4 to 3 places of decimals.
10. „ ·0045 by ·03.

Surveyor of Taxes.

11. Divide 327·6 by 4·89 to 3 places of decimals.
12. „ 55·8 by ·08125 to 3 places of decimals.
13. „ 12 by ·0009.

---

* The learner should omit Nos. 6, 7, 13, 15, 17, 20, 21, 32, 33, 34, 37, 38, and 43 until he has read Circulators.

Various Departments.

14. Divide 274·6 by 3·672 to 3 places of decimals.
15. „ 89·2 by ·0074 to 3 places of decimals.
16. „ 16 by ·0004.
17. „ 21546·872 by ·054.
18. „ 34·952 by 53·07 giving the first 4 figures of the quotient.
19. „ ·1 by ·001 and the quotient by 2.
20. „ 721·42 by 21·9 to 4 places of decimals.
21. „ ·169 by 13, by 130, and by ·0013.
22. „ 91·6 by 8931·61 to 4 places of decimals.
23. „ 43·2 by ·0351 to 3 places of decimals.

94. *To reduce a decimal to its equivalent vulgar fraction.*

Rule.—Omit the point, and write the figures of the decimal as numerator, and 1 with as many cyphers as there are decimal places for denominator.

Thus, $\cdot755=\frac{755}{1000}=\frac{151}{200}$; also, $\cdot8=\frac{8}{10}=\frac{4}{5}$; also $\cdot025=\frac{25}{1000}=\frac{5}{200}=\frac{1}{40}$.

This is merely supplying the denominator which is suppressed in every decimal.

Various Departments.

24. Express ·0125 as a vulgar fraction in its lowest terms.
25. Express ·00175 as a vulgar fraction in its lowest terms.
26. Reduce 2·0445 and ·000625 to vulgar fractions.
27. Reduce 23·038125 and ·0006875 to vulgar fractions.

95. *To reduce a vulgar fraction to a decimal.*

Rule.—Divide the numerator by the denominator according to the rule for the division of decimals. The quotient will be the required decimal.

Thus, $\frac{3}{4}=3\div4=\cdot75$; and $\frac{3}{8}=3\div8=\cdot375$.

Reason.—We require to bring $\frac{3}{4}$ to a decimal—one whose denominator is 10, or some power of 10; a method of doing which was given (**66**). By the rule referred to, we find, as 4 measures 100, the multiplier is 25, for any fraction whose denominator is 4; therefore $\frac{3}{4}\times\frac{25}{25}=\frac{75}{100}=\cdot75$, the same result as found above. But multiplying by 25 is the same as adding 00 and dividing by 4; hence the rule.

It follows that a fraction whose denominator is *not* a factor or measure of 10 or some power of 10, cannot be reduced to an equivalent decimal *exactly*.

Reduce to decimals:—

28. $\frac{1}{2}$, $\frac{1}{5}$, $\frac{7}{8}$.

29. $\frac{7}{16}$, $\frac{11}{25}$, $\frac{13}{16}$.

96. ***To reduce one quantity to the decimal of another of the same kind.***

Rule.—Express the former as a fraction of the latter and reduce this fraction to a decimal.

Ex. 1. Reduce 12*s*. 6¾*d*. to the decimal of a pound?

$$\text{Here, } \frac{12s.\ 6\frac{3}{4}d.}{£1} = \frac{150\frac{3}{4}d.}{240d.} = \frac{603}{960} = \frac{201}{320} = \cdot628125 \textit{ Ans.}$$

Reduce to the decimal of a *pound*:—

30. 15*s*. 9¾*d*., 17*s*. 9*d*.

XXIV.

*Various Departments.**

31. Reduce 8*s*. 1½*d*. to the decimal of a £.
32. „ 6⅔ cwts. to the decimal of a ton.
33. „ ⅝ of a crown to the decimal of 21*s*.
34. „ 1·85 of 3*s*. 4*d*. to the decimal of a guinea.

Admiralty.

35. Reduce 12 cwts. 1 qr. 7 lbs. to the decimal of a ton.

Various Departments.

36. Reduce $\frac{3}{32}$ to a decimal.
37. „ $\frac{40}{33}$ to a decimal.
38. Reduce $\frac{5}{64}$ and $\frac{3}{14}$ to decimals.
39. „ $\frac{21}{250}$ to a decimal.

97. ***To reduce any number of shillings, pence and farthings, accurately to the decimal of a pound, at once.***

Rule.—1°. Conceive a 0 to be annexed to the shillings, and take one-half that number for the first part of the decimal.

---

* The chapter on Circulators may be now read with advantage.

2°. Falling back one place to the right, write the number of farthings in the pence and farthings.

3°. Falling back another place to the right, set down one-sixth of the pence and farthings, considering the farthings expressed as the decimal of a penny.

4°. Then add these three lines of figures for the decimal required.*

Ex. 1. Reduce 13*s.* 9¾*d.* to the decimal of a pound.

Here we take one-half of 130 = 65; then 9¾*d.* = 39*f.*, the next part; and 9¾*d.* ÷ 6 = 9·75*d.* ÷ 6 = 1·625, the last part.

13*s.* 9¾*d.*
·65
39
1625
·690625£. *Ans.*

Ex. 2. Reduce 19*s.* 4¾*d.* to the decimal of a pound.

Here we take one-half of 190 (a 0 annexed to the shillings) = ·95; next, we write the number of farthings in 4¾*d.* = 19; and lastly, we write one-sixth of 4·75 (4¾*d.*) = ·07916̇.

19*s.* 4¾*d.*
·95
19
07916̇
·9697916̇£. *Ans.*

Reduce to the decimal of a pound:—

	s.	*d.*		*s.*	*d.*
40.	17	6,	and	13	7½
41.	16	9¾,	and	19	6
42.	17	9¾,	and	7	11¼
43.	10	10½,	and	4	4½

98. *To find the value of the decimal of a given quantity.*

RULE.—1°. Multiply the decimal, as in reduction, by the number of units of the next lower denomination which makes one of the given denomination.

---

* Since 1*s.* = £$\frac{1}{20}$, 1*s.* = £·05; and 2*s.* = ·1: hence the reason of taking half the shillings. Adding the 0 is unnecessary when the number of shillings is even; for *one* shilling write ·0 before dividing by 2. Again, £1 = 960*f.*, which wants 40 of being 1000, the number of thousandths in a unit (in a £); 40 wanted in 960 is $\frac{40}{960}=\frac{1}{24}$, that is, 1 in 24; in other words, there will be more thousandths than farthings by 1 in every 24*f.*, or 1 in every 6*d.*; hence the reason of adding ⅙ of the pence. Conversely, there will be *less* farthings than thousandths by 1 in 25, for there are 40 *less* in 1000, and $\frac{40}{1000}=\frac{1}{25}$; hence the decimals may be considered farthings, after rejecting 1 in every 25.

2°. The whole number (if any) thus obtained will be in the lower denomination.

3°. Multiply the decimal part in the same manner, for the next lower denomination, and so on.

Ex. 1. Find the value of ·675 of a *cwt.*

Here we multiply by 4, and find 2 a whole number, which is 2 qrs.; again, we multiply by 28 and find 19·6 lbs., and so on.

$$\begin{array}{rl} & \text{cwt.} \\ & ·675 \\ & 4 \\ \hline 2·700 & \text{qrs.} \\ 28 & \\ \hline 19·600 & \text{lbs.} \\ 16 & \\ \hline 9·6 & \text{oz.} \\ 16 & \\ \hline 9·6 & \text{drs.} \end{array}$$

2 qrs. 19 lbs. 9 oz. 9·6 drs. *Ans.*

Ex. 2. Find the value of ·526 of £1 10*s.*

Here, ·526 × 30 = 15·78*s.*; and ·78 × 12 = 9·36*d.*; then ·36 × 4 = 1·44: therefore 15*s.* 9¼*d.* *Ans.*

Find the value of: —

44. ·7689£., ·465*s.*

45. ·976 days.

46. ·634 of 6*s.* 8*d.*

47. ·7958 of an acre.

99. *To find the value of any decimal of a pound at once.*

Rule.—1°. Take the first three figures of the decimal; increase the third by 1, if the fourth decimal figure be greater than 5.

2°. Then, take one-fifth of the number expressed by the first two figures for shillings.

3°. Prefix the remainder (if any) to the third, and after rejecting one twenty-fifth, consider the result as farthings.

Ex. 1. Find the value of ·7967£.

As the fourth figure is 7, we add 1 to the third; then we have ·797; now, 79 ÷ 5 = 15*s.* + 4r., and 47 − 2 = 45*f.* = 11¼*d.* We subtract 2 because 47 is nearly 50; hence 15*s.* 11¼*d.* *Ans.*

Reason.—Considering the first and second figures as a whole number is the same as multiplying by 100; and we divide this by 5. Since 100 ÷ 5 = 20, multiplying by 100, and dividing by 5, is equivalent to multiplying by 20, the number of shillings in a pound. (*See* **97** *and note.*)

Find the value of:—

48. ·6583£., and of ·97832£.

Customs and Excise.

49. Add the following and find the value of it in £. s. d.

$$\begin{array}{r} 1·521 \\ ·675 \\ ·145 \\ 3·184 \\ \hline \end{array}$$

50. Add together ·0204£., 1·326£., and ·003£., and find the value of the sum.

Admiralty.

51. Find the value of ·75 of a £.

52. What is the value in money of ·25 of a £.?

53. What is the value in money of ·73125 of a £.?

54. What is the value in money of ·025 of a £.?

Various Departments.

55. What is the value of ·4375 of a shilling?

CIRCULATING DECIMALS.

100. One fraction can be reduced to another equal in value and having *any denominator*, when the denominator of the given fraction is a *factor* of that to which it is to be reduced (**66.**) Now, as the denominator of every decimal is 10 or some power of 10, we cannot reduce those vulgar fractions to equivalent decimals which have denominators that are *not* factors of 10, or some power of 10. In the reduction of such fractions to decimals, the division does not terminate, and the same figures will be repeated over again, continually. If we try to reduce $\frac{2}{3}$ to a decimal, it becomes

$\frac{2}{3} = \cdot 6666$ &c.; also, $\frac{5}{9} = \cdot 5555$ &c. Decimals of this kind, in which the same figures are continually repeated, are called *Repeating*, *Recurring*, or *Circulating* decimals. The part repeated is called the *Period* or *Repetend*, and is termed a *Simple Repetend* when it consists of one figure; when it consists of more than one figure it is called a *Compound Repetend*. Decimals, arising from vulgar fractions, whose denominators give *exact* quotients, are called *Terminate*. The period of a circulator is marked by means of dots or accents, placed over its first and last figures. When only one figure is repeated, one dot is sufficient. Thus, $\frac{2}{3} = \cdot\dot{6}$, and $\frac{1}{9} = \cdot\dot{1}$.

When the period begins *immediately* after the point, it is called a *Pure Circulator;* as, $\frac{1}{3} = \cdot\dot{3}$.

A *Mixed Circulator* has a finite and an infinite, or circulating part. Thus, $\frac{1}{6} = \cdot 1\dot{6}$. The finite part is ·1, and the recurring part is $\cdot\dot{6}$.

We find by using the nine figures as denominators and 1 as numerator, that 2, 4, 5, and 8, give *terminate;* 3, 7, 9, *pure circulators;* and 6, a *mixed circulator*, as in the annexed equations.

$\frac{1}{2} = \cdot 5$
$\frac{1}{3} = \cdot\dot{3}$
$\frac{1}{4} = \cdot 25$
$\frac{1}{5} = \cdot 2$
$\frac{1}{6} = \cdot 1\dot{6}$
$\frac{1}{7} = \cdot\dot{1}4285\dot{7}$
$\frac{1}{8} = \cdot 125$
$\frac{1}{9} = \cdot\dot{1}$

101. Any fraction in its lowest terms whose denominator contains only powers of 2 and 5 as factors, can be reduced to a terminate decimal.

Reason.—A fraction brought to its *lowest terms* can have no factor common to both numerator and denominator: therefore it is in consequence of the *addition* of cyphers to the numerator, that is, the multiplication by 10 or some power of 10, that the denominator measures the numerator; in other words, it is the number by which the numerator is multiplied that the denominator measures. But 10 contains only the factors 2 and 5; any power of 10 will only contain powers of 2 and 5. Therefore the denominator, to measure any power of 10, must be made up of 2 and 5, or powers of these factors. If the denominator be so made up, the decimal will terminate; but if the denominator be made up of any other factors, as 3, 7, 9, &c., it will produce a *circulator*.

102. The number of figures in a period must always be less than the number of units in the denominator.

Because different figures must arise from different remainders, and the number of remainders must be less than the divisor or denominator. Thus, $\frac{1}{7} = \dot{1}4285\dot{7}$. When dividing by 7, we can only have 1, 2, 3, 4, 5, 6, as remainders. In dividing by any number, the greatest remainder must be less than the divisor.

103. *To reduce a pure circulator to a vulgar fraction.*

Rule. Write the period for numerator, and so many nines as there are figures in it, for denominator. Thus, $\cdot\dot{3} = \frac{3}{9} = \frac{1}{3}$; and $\cdot\dot{2}\dot{7} = \frac{27}{99} = \frac{9}{33} = \frac{3}{11}$.*

Reason.—Since $\frac{1}{3} = \cdot3333$, &c., multiply by 10, and we have ten-thirds $= 3\cdot333$, &c.; then, subtracting $\cdot333$, &c., from $3\cdot333$, &c., we get 3, which must be equal to nine-thirds, as we took the value of $\frac{1}{3}$ from ten-thirds. Now, since $\frac{9}{3} = 3$, one-third must equal $\frac{3}{9}$, therefore $\cdot333$, &c., $= \frac{3}{9}$.

XXV.

Reduce to equivalent vulgar fractions:—

1. $\cdot\dot{7}$, and $\cdot\dot{6}$.
2. $\cdot\dot{4}\dot{5}$, and $\cdot\dot{1}3\dot{5}$.

104. *To reduce a mixed circulator to a vulgar fraction.*

Rule.—1°. From the finite part and period, considered as one number, subtract the finite part; the difference will be the numerator.

2°. For the denominator, write as many nines as there are figures in the period, followed by as many cyphers as there are figures in the finite part. Thus, $\cdot1\dot{6} = 16 - 1 = 15$, and $\frac{15}{90} = \frac{3}{18} = \frac{1}{6}$.†

Proof.—$\frac{1}{6} = \cdot1\dot{6}$.

---

* $\frac{1}{9} = \cdot1111$, &c.; $\frac{2}{9} = \cdot2222$, &c.; $\frac{5}{9} = \cdot55\dot{5}$ &c. Also, $\frac{1}{99} = \cdot010101$, &c.; and $\frac{15}{99} = \cdot1515$, &c.: therefore any figure divided by 9 gives that same figure as period, and any two figures divided by 99 will give those *two figures* as period; hence the reason of the rule.

† Take $\frac{7}{30}$, which is $= \cdot2\dot{3}$; multiply this mixed circulator by 10, and we get $2\cdot\dot{3}$, which is $2\frac{3}{9}$ (**103**). Now, $2\frac{3}{9}$ is 10 times too great, as we multiplied by 10; then dividing by 10 we get $\frac{2}{10} + \frac{3}{90}$ (dividing both separately). By adding, we get $\frac{2\times90}{900} + \frac{30}{900} = \frac{2\times9}{90} + \frac{3}{90}$ and since $9 = 10 - 1$, we have $\frac{2\times(10-1)}{90} + \frac{3}{90} = \frac{2\times10-2}{90} + \frac{3}{90}$; adding numerators, we have $\frac{20+3-2}{90} = \frac{23-2}{90} = \frac{21}{90} = \frac{7}{30}$ which is the rule.

Reason.—Multiply $\cdot 1\dot{6}$ by 10, and we have ... ... $1\cdot\dot{6}$ *Take*
Again, multiply $\cdot 1\dot{6}$ by 100, and we have ... ... $16\cdot\dot{6}$ *From*
Subtracting the former product from the latter we get 90 times $\cdot 1\dot{6} = 15$; therefore $\cdot 1\dot{6} = \frac{15}{90}$, which is the rule. $\overline{15}$

Reduce to vulgar fractions:—

3. $\cdot 2\dot{7}$, $\cdot 5\dot{9}\dot{0}$.
4. $\cdot 58\dot{3}$, $\cdot\dot{6}9230\dot{7}$.
5. $\cdot 7\dot{3}$ and $\cdot 8\dot{3}$.
6. $\cdot\dot{1}\dot{8}$ and $\cdot 791\dot{6}$.

National Education (Ireland.)

7. Multiply $\cdot 0\dot{0}2\dot{1}$ by $48\cdot 02\dot{6}$.

105. Since (**103.**) $\cdot\dot{3} = \frac{3}{9}$, it follows that $\cdot\dot{9} = \frac{9}{9} = 1$. It is evident that we cannot reverse this operation and bring 1 to ·99, &c.; but if we repeat the period we shall find that it approaches so near 1, that the difference will be absolutely nothing; for $\cdot 999 + \frac{1}{1000} = 1$, and $\cdot 9999 + \frac{1}{10000} = 1$, &c. Therefore when we meet $\cdot\dot{9}$ in any decimal we may add 1 to the preceding figure, and say, $\cdot 6\dot{9}\dot{9}$, &c., $= \cdot 7$; $\cdot 4\dot{9} = \cdot 5$; and $\cdot 7\dot{9} = \cdot 8$.

106. When *great* accuracy is required, circulators should be reduced to vulgar fractions; but, in general, it will be sufficient for practical purposes to repeat the period to *three* or *four* places.

To add $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$.

Here, we bring the circulators to four decimal places. We find the answer by decimals, 1·4499, &c., which may be considered $= 1\cdot 45$; and $1\cdot 45$ is $= 1\frac{45}{100} = 1\frac{9}{20}$. Or, $1\cdot 449\dot{9} = 1\frac{4499-44}{9900} = 1\frac{4455}{9900} = 1\frac{891}{1980} = 1\frac{99}{220} = 1\frac{9}{20}$, as before.

Vulgar Fractions.	*Decimals.*
$\frac{1}{2} = 30$ (60)	$\frac{1}{2} = \cdot 5$
$\frac{1}{3} = 20$	$\frac{1}{3} = \cdot 3333$; &c.
$\frac{1}{4} = 15$	$\frac{1}{4} = \cdot 25$
$\frac{1}{5} = 12$	$\frac{1}{5} = \cdot 2$
$\frac{1}{6} = 10$	$\frac{1}{6} = \cdot 1666$, &c.
$\frac{87}{60} = 1\frac{27}{60} = 1\frac{9}{20}$ *Ans.*	1·4499, &c. *Ans.*

The following will further illustrate what has been said:—

$\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$; now, $\frac{1}{3} = \cdot 333$, &c., and $\frac{1}{6} = \cdot 166$, &c.; then $\cdot 333 \times \cdot 166 = \cdot 055278$, the product of $\frac{1}{3}$ and $\frac{1}{6}$ when reduced to *circulators* of *three* figures; but $\frac{1}{18} = \cdot 0555$, &c.; the two products only coincide in three figures. The number of decimal places to be used will depend very much on the nature of the case. If we have three true places in any result, it is sufficient in commercial matters. For, since £1 = ·960*f.*, 1*f.* is greater than $\frac{1}{1000}$ (·001). If the fourth place were 9, it would be much less than a farthing.*

107. From the preceding observations it is evident that the kind of decimal any vulgar fraction will give, can be determined from its denominator in its lowest terms; if a mixed circulator, the number of figures in its finite part; and if a finite decimal, the number of decimal places. Any *pure circulator* must have so many figures in its period as will be equal to the least number of *nines* which will be measured by its denominator. The denominator must be a factor of a certain number of nines, (**66.**)

A *mixed circulator* arises from a denominator made up of either *twos* or *fives* and some other factors. Thus, 6 is $= 3 \times 2$; here 2 will give a *finite* part, and 3 an *infinite* part.

Again, take $\frac{23}{54}$; now $54 = 2 \times 3 \times 3 \times 3$; hence any vulgar fraction having 54 as denominator, will give a decimal having one figure in its finite part, corresponding to the factor 2, and three figures in its period corresponding to $3 \times 3 \times 3$. We find $\frac{13}{54} = \cdot 2\dot{4}0\dot{7}$. This is true irrespective of the numerator; for $\frac{2}{54} = \cdot 0\dot{1}8\dot{5}$, and $\frac{53}{54} = \cdot 9\dot{8}1\dot{4}$.

A *Finite* decimal will have as many decimal places as are equal to the greatest number of *twos* or *fives* in the factors of its denominator in its lowest terms. If different denominators be compared with the decimals they produce, the reason will be evident.

From the previous explanations and definitions it is evident that all numbers will *measure* a number made up of 1 followed by one or more cyphers; a number formed of one or more nines; or of one or more nines followed by one or more cyphers.

---

* When we have *two* decimal places in any result, the *error* by omitting any other decimal figures, must be less than $\frac{1}{100}$, for ·009 is less than ·01. If we have *three* places of decimals, the *error* must be less than $\frac{1}{1000}$, for ·001 is greater than ·0009; this is called *the Limit of the Error*; and since $\frac{1}{1000}$ is less than 1*f.*, three true decimal places in any result will be quite sufficient for mercantile calculations.

MISCELLANEOUS EXERCISES IN FRACTIONS.

XXVI.

Customs, Excise, &c.

Add the following and bring them to their proper values:—

£	Tons.
1. 4785·321	2. 576·098
547·9	88·47
643·198	694·321
599·97	745·879
654·099	33·4
9·9	7·9
584·498	654·54
337·87	89·785
4679·3	547·678
944·32	99·785

3. Multiply the following and bring them to their proper values :— £54·32 by 4·7.

4. and 5987 cwts. by ·99.

5. From $\frac{5}{8}$ take $\frac{2}{5}$ of $\frac{5}{8}$.

Constabulary and Education Offices.

6. Divide $34\frac{1}{2}$ by $5\frac{3}{4}$.

7. Divide 91·4 by 9020·4 to 4 places of decimals.

8. What number added to $1\frac{7}{10}$, $3\frac{9}{16}$, $2\frac{1}{26}$, $\frac{5}{24}$, will make the sum total 10?

9. Reduce 3·45 of half-a-guinea to the decimal of half-a-crown.

10. Reduce 3·285 of 6*s.* 8*d.* to the decimal of a guinea.

11. Divide 39·49 by 13·476 to 4 places of decimals.

12. Divide $19\frac{2}{7}$ by $4\frac{3}{8}$.

13. Find the value of 2·625 of a £.

War Department.

14. Find the greatest common measure of 3042 and 3094 and reduce $\frac{3042}{3094}$ to its lowest terms.

The following have been proposed in Various Departments.

15. Add together $\frac{3}{5}$, $\frac{4}{9}$, $\frac{3}{10}$, and $\frac{7}{18}$, and divide the result by $7\frac{5}{8}$.

16. Find the value of $\frac{2}{3}$ of $\frac{5}{8}+\frac{3}{7}$ of $\frac{14}{15}$ of a shilling.

17. Reduce $\frac{23205}{46436}$ to its lowest terms.

18. Add together $\frac{2}{3}+\frac{1}{5}+\frac{2}{15}+1\frac{3}{7}+\frac{4}{105}$ and subtract the result from $3\frac{1}{15}$.

19. Multiply 21·5839 by ·00158.

20. Reduce £8 9*s.* 3*d.* to a fraction of £21 12*s.* 9*d.*, expressing the result in its lowest terms.

21. Reduce $\frac{1}{7}$ of $2\frac{1}{3}$ of $6\frac{7}{8}\div4\frac{2}{5}$ to a simple fraction, and multiply the result by $1\frac{23}{25}$.

22. Add together $\frac{5}{12}$ of a pound, $\frac{3}{7}$ of a guinea, $\frac{4}{13}$ of a shilling, and $\frac{8}{13}$ of a half-penny.

23. Reduce $\frac{1}{13}$ to a decimal showing the whole of the circulating period.

24. Reduce $\cdot13\dot{0}\dot{9}$ to a vulgar fraction in its lowest terms.

25. Divide 69·814 by ·00521 and by 52100.

26. Reduce $\frac{3}{4}$ of half-a-crown to the decimal of a £.

27. Find the value of ·8473 of a cwt.

28. Subtract 423·79283 from 1857·23.

29. What number added to $3\frac{2}{3}$, $1\frac{9}{20}$, $2\frac{7}{12}$, $1\frac{8}{15}$, will make the sum total 12?

30. Add together $\frac{5}{13}$ of a pound, $\frac{3}{16}$ of 6*s.* 8*d.*, $\frac{1}{40}$ of a crown, and $\frac{9}{13}$ of a penny.

31. What number multiplied by $\frac{2}{5}$ will give $15\frac{3}{8}$ as the product?

32. Add together $3\frac{5}{8}$ pounds, $9\frac{3}{7}$ shillings, and $2\frac{4}{5}$ pence.

33. Add together ·130055, 900, ·57·1, 13·34, and ·00000397 and subtract the result from 1314·9.

34. Multiply 95·376 by ·0002.

35. Express $\frac{241}{15424}$ as a decimal.

36. Reduce $\frac{7}{8}$ of half-a-crown to the decimal of a guinea.

37. Reduce $\cdot18\dot{1}5\dot{3}$ to a vulgar fraction in its lowest terms.

38. Express 7*s.* $8\frac{1}{2}$*d.* as the decimal of a £.

39. Find the value of ·03234 of a guinea.

40. Divide ·18 by ·004.

41. Divide $12\frac{5}{9}$ by $9\frac{2}{3}$.

42. Divide 738·952716 by ·07 and by 70.

43. Express 7*s.* $10\frac{1}{2}$*d.* as the decimal of a pound.

44. Multiply $\dot{3}$ by $\dot{2}$ and express the result as a circulating decimal.

War Department.

45. Add sixty and three thousandths to thirty-three and sixty-five hundred thousandths.

46. Subtract the less from the greater.

47. Find their product.

48. Divide the greater number by the less.

PRACTICE.

108. Practice is the process of finding the value of a quantity by means of aliquot parts when the price of a unit of any denomination of that quantity is given.

An ALIQUOT PART of a quantity or number is one which when taken a certain number of times makes up that quantity or number exactly. 4 is an aliquot part of 8, because 4 when taken twice makes up 8 *exactly*; 6*d.* is an *aliquot part* of a shilling, or of a pound, because when taken twice it is $=1$*s.*, and when taken 40 times it is $=$ £1 *exactly*. We say 6*d.* $= \frac{1}{2}$ of a shilling; 6*d.* $= \frac{1}{40}$ of a pound, and so on.

To find the *aliquot parts* of any quantity or number, divide by 2, 3, 4, &c., rejecting any divisors which produce *fractions* in the quotient.

ALIQUOT PARTS OF £1.

s.	*d.*		*s.*	*d.*		*d.*		*d.*	
10	0	$= \frac{1}{2}$	2	6	$= \frac{1}{8}$	10	$= \frac{1}{24}$	4	$= \frac{1}{60}$
6	8	$= \frac{1}{3}$	2	0	$= \frac{1}{10}$	8	$= \frac{1}{30}$	2	$= \frac{1}{120}$
5	0	$= \frac{1}{4}$	1	8	$= \frac{1}{12}$	$7\frac{1}{2}$	$= \frac{1}{32}$	$1\frac{1}{2}$	$= \frac{1}{160}$
4	0	$= \frac{1}{5}$	1	3	$= \frac{1}{16}$	6	$= \frac{1}{40}$	1	$= \frac{1}{240}$
3	4	$= \frac{1}{6}$	1	0	$= \frac{1}{20}$	5	$= \frac{1}{48}$	$\frac{1}{2}$	$= \frac{1}{480}$

CASE I.

109. *When the price and quantity are each of only one denomination.*

RULE.—The product of the price and quantity will be the answer in the *same name* as the price.

Ex. 1. 324 yards of cloth, at 7*s.* per yard.

Here, 324 × 7 = 2268*s.* = £113 8*s.* *Ans.*[*]

REASON.—324 yards, at a shilling per yard, will cost 324*s.*; therefore at 7*s.* the cost will be 7 *times* as much.

110. *If the price be an aliquot part of a higher denomination of money.*

RULE.—Take a like part of the quantity for the *answer.*

Ex. 2. Find the price of 324 yards, at 5*s.* per yard.

Here, 5*s.* = $\frac{1}{4}$ of a pound; every 4 yards will cost a pound; hence we take one-fourth of £324 for the answer.

yds.
324 at 5*s.*
5*s.* = $\frac{1}{4}$ = £81 *Ans.*

Ex. 3. Find the value of 326 yards, at 4*d.* per yard.

Here, 4*d.* = $\frac{1}{3}$ of a shilling; the price of every 3 yards being a shilling, we take the third part; we have 2 remaining; as each yard costs 4*d.*, two yards will cost 8*d.*, that is $\frac{1}{3}$ of 2*s.*

yds
326 at 4*d.*
d.
4 = $\frac{1}{3}$ = 10,8*s.* 8*d.*
$\frac{1}{20}$ = £5 8*s.* 8*d.* *Ans.*

Post Office.

Ex. 4. Find the value of 653 articles at $\frac{3}{4}$*d.* each.

In this example we multiply the number of articles by $\frac{3}{4}$*d.*, and have the answer in farthings, which we reduce to £ *s.* *d.*

653 at $\frac{3}{4}$*d.*
3
4)1959 farthings.
12)489 $\frac{3}{4}$*d.*
2,0)4,0 9$\frac{3}{4}$
£2 0 9$\frac{3}{4}$ *Ans.*

By First Principles.—Cost of 653 at $\frac{1}{4}$*d.* = 653*f.* = 13*s.* 7$\frac{1}{4}$*d.* At $\frac{3}{4}$*d.* it will be three times as much; then 13*s.* 7$\frac{1}{4}$*d.* × 3 = £2 0*s.* 9$\frac{3}{4}$*d.* as before.

---

[*] Strictly speaking, this operation is not Practice; but, as it is better always to select the most convenient method (which this frequently is), it has been inserted. In fact, all the other rules given under Practice are modifications of this one.

Ex. 5. Find the value of 1245 things at 4*d.* each.

At 1*s.* each 1245 will be 1245*s.*, and at 4*d.* each, since 4*d.* = $\frac{1}{3}$ of a shilling, the value will only be one third as much.

1245 @ 4*d.*
4*d.* = $\frac{1}{3}$ of a *s.* = 415*s.* = £20 15*s.* *Ans.*

By First Principles.—Cost of 1245 at 1*d.* each = 1245*d.* = £5 3*s.* 9*d.*; hence, at 4*d.*, it will be £5 3*s.* 9*d.* × 4 = £20 15*s.* as before.

XXVII.

Post Office.

1. Find the value of 846 articles at $\frac{3}{4}$*d.* each.
2. „ 352 „ 1$\frac{1}{2}$*d.* „
3. „ 729 „ 11*d.* „
4. „ 89379 „ $\frac{1}{4}$*d.* „
5. „ 979 „ 9*d.* „

CASE II.

111. *When the quantity is of one denomination and the price of several denominations.*

RULE.—1°. Multiply the quantity by the highest denomination in the price.

2°. Take aliquot parts of *one* of this highest denomination and add the results.

Post Office.

Ex. 6. Find the value of 144 reams of paper at 13*s.* 4*d.* per ream.

144 @ 13*s.* 4*d.*
13
1872*s.* value at 13*s.*
4*d.* = $\frac{1}{3}$ of a *s.* = 48*s.* „ at 4*d.*
1920*s.* = £96 *Ans.*

By First Principles.

Cost of 144 at 1*s.* per ream . . . . . = 144*s.* = £7 4
„ 144 at 13*s.* „ . . . . . = £7 4 × 13 = 93 12
„ 144 at 1*d.* = 12*s.*, and at 4*d.* = 12*s.* × 4 = 2 8
£96 0*s.* *Ans.*

Or by the principle explained (**31**) divide one and multiply the other by 3, and we have $\frac{144}{3}$ @ 13*s.* 4*d.* × 3 = 48 @ £2 = £96.

Various Departments.

Ex. 7. What is the dividend on £2734 16*s.* 8*d.* @ 9*s.* 4½*d.* in the £?

$$\begin{array}{rrl}
 & 2734 & @\ 9s.\ 4\tfrac{1}{2}d. \\
 & 9 & \\ \hline
 & 24606 & \\
4d.=\frac{1}{3}\text{ of a }s.= & 911 & 4 \\
\frac{1}{2}d.=\frac{1}{8}\text{ of }4d.= & 113 & 11 \\ \hline
 & 25631 & 3 \\
16s.\ 8d.= & 7 & 9\frac{3}{4} \\ \hline
 & 2,0)2563,9 & 0\frac{3}{4} \\ \hline
\end{array}$$

$$\begin{array}{rrr}
 & s. & d. \\
 & 9 & 4\frac{1}{2} \\
s. & & \\ \hline
10=\frac{1}{2}= & 4 & 8\frac{1}{4} \\
6s.\ 8d.=\frac{1}{3}= & 3 & 1\frac{1}{2} \\ \hline
 & 7 & 9\frac{3}{4}
\end{array}$$

£1281 19*s.* 0¾*d.* *Ans.*

By applying the principle explained in (31) we have £2734 16*s.* 8*d.* × 6 = £16409; then 9*s.* 4½*d.* ÷ 6 = 1*s.* 6¾*d.*; whence we have—

$$\begin{array}{rrl}
 & 16409 & @\ 1s.\ 6\tfrac{3}{4}d. \\
6d.=\frac{1}{2}= & 8204 & 6 \\
\frac{3}{4}=\frac{1}{8}= & 1025 & 6\frac{3}{4} \\ \hline
 & 2,0)2563,9 & 0\frac{3}{4} \\ \hline
\end{array}$$

£1281 · 19*s.* 0¾*d.* *Ans.*

Post Office.

6. **Find the value of 3107 sheep, each sheep being worth £1 14*s.* 7½*d.***
7. **Find the dividend on £3762 10*s.* at 8*s.* 2½*d.* in the pound.**

Various Departments.

8. **Find the value of 204 at £3 12*s.* 5*d.***

Admiralty Office.

9. **Find the value of 4562¼ things at £3 15*s.* 9½*d.* each.**

Various Departments.

10. **Find the value of 6723 pieces of cloth, each piece being worth £1 8*s.* 8¼*d.***
11. **What is the dividend on £2468 at 15*s.* 6*d.* in the £ ?**
12. **Find the value of 373 articles at 9*s.* 7¾*d.* each.**
13. **" 875 " at 5*s.* 11*d.* each.**
14. **" 327 " at £1 2*s.* 5½*d.***
15. **" 373 " at £2 16*s.* 10¼*d.***
16. **" 217¼ yards of lace at £2 17*s.* 7½*d.* each.**
17. **" 136 articles at £1 7*s.* 2*d.***
18. **" 372 " at 3*s.* 7½*d.***
19. **" 204 " at £3 12*s.* 5*d.***

Admiralty Office.

20. Find the value of 3875 articles at £8 18*s.* 6¾*d.*
21. „ 7394¾ „ at £12 8*s.* 8¼*d.*

Various Departments.

22. Find the value of 27 cwts. of sugar at £2 18*s.* per cwt.
23. „ 473 lbs. of coffee at 1*s.* 5½*d.* per lb.
24. „ 132½ tons of iron at £7 15*s.* 6*d.* per ton.
25. Find the cost of 716 acres at £44 11*s.* 3¼*d.* per acre.
26. Find the value of 6214 sheep, each sheep being worth £1 14*s.* 7½*d.*

CASE III.

112. *When both the commodity and price are of several denominations.*

RULE.—Multiply the price by the highest denomination in the quantity, and take aliquot parts of the price for the remainder of the quantity.

Prisons, Science and Art, &c.

Ex. 8. Find the price of 13 cwts. 3 qrs. 11 lbs. of sugar at £2 6*s.* 8*d.* per cwt.

The cost of 13 cwts. is found by multiplying £2 6*s.* 8*d.* by 13. Then aliquot parts are taken for the 3 qrs. 11 lbs., as will be clearly understood from the annexed work.

cwts. qrs. lbs.			
13 3 11 @	£2	6	8
			13
	30	6	8
2 qrs. = ½ of a cwt. =	1	3	4
1 qr. = ½ of 2 qrs. =		11	8
7 lbs. = ¼ of 1 qr. =		2	11
4 lbs. = ⅐ of 1 qr. =		1	8
	£32	6	3 *Ans.*

27. Find the price of 17 acres 3 rds. 15 pls. at £37 10*s.* per acre.

Various Departments.

28. Find the price of 19 cwts. 1 qr. 13 lbs. at £2 11*s.* 4*d.* per cwt.
29. Find the value of 39 cwts. 1 qr. 20 lbs. at £6 2*s.* 6*d.* per cwt.

Admiralty.

30. Find the value of 38 cwts. 1 qr. 16 lbs. at £2 12*s.* 6*d.* per cwt.
31. Find the value of 16 cwts. 1 qr. 2 lbs. at £3 13*s.* 9*d.* per cwt.

Various Departments.

32. Find the value of 13 cwts. 3 qrs. 17 lbs. of tea at £22 8*s.* per cwt.

33. Find the cost of 14 cwts. 3 qrs. 19 lbs. at £5 16*s.* 8*d.* per cwt.

113. If the price be not that of a *unit* of the highest denomination, reduce the given quantity to that denomination of which the price is given. Or, find from the given price, what the price of *one* of the highest denomination would be at the same rate.

Post Office.

Ex. 9. Find the price of 3 cwts. 2 qrs. 17 lbs. at £1 5*s.* 8*d.* per quarter.

It is evident that £1 5*s.* 8*d.* × 4 = £5 2*s.* 8*d.* the price per cwt. at same rate; hence,

3 cwts. 2 qrs. 17 lbs. @	£5	2	8
			3
cost of 3 cwts. =	15	8	0
2 qrs. = $\frac{1}{2}$ of 1 cwt. =	2	11	4
14 lbs. = $\frac{1}{4}$ of 2 qrs. =	0	12	10
2 lbs. = $\frac{1}{7}$ of 14 lbs. =	0	1	10
1 lb. = $\frac{1}{2}$ of 2 lbs. =	0	0	11
	£18	14	11 *Ans.*

Or since 3 cwts. 2 qrs. 17 lbs. = 14 qrs. 17 lbs., we have

	£1	5	8
			14
price of 14 qrs. =	17	19	4
14 lbs. = $\frac{1}{2}$ of a qr. =	0	12	10
2 lbs. = $\frac{1}{7}$ of 14 lbs. =	0	1	10
1 lb. = $\frac{1}{2}$ of 2 lbs. =	0	0	11
	£18	14	11 *Ans.*

Customs and Excise.

34. Find the value of 7 cwts. 3 qrs. 11 lbs. at £2 13*s.* 1*d.* per quarter.

Constabulary (Cadetships).

35. Find (by Practice) the price of 8 tons 13 cwts. 18 lbs. of coal at $3\frac{1}{2}$*d.* per quarter.

36. Find the dividend on £1760 5*s.* at 10*s.* 4*d.* in the £.

Post Office, Board of Trade, Fisheries, &c.

37. A rate of 1*s*. 5*d*. in the pound is levied on a parish where the rateable rental is £360817 10*s*. Find the amount produced.

38. Find the value of 105 pockets of hops, each weighing 1 cwt. 1 qr. 8 lbs., at £4 12*s*. 6*d*. per cwt.

CONTRACTIONS.

114. *For tons, cwts., qrs.*

RULE.—Multiply the tons by 1, and consider the product as pounds; the cwts. by 1, and consider the product as shillings; the qrs. by 3, and consider the product as pence.

£1 per ton is 1*s*. per cwt. and 3*d*. per qr.: hence the rule.

Various Departments.

Ex. 10. Find the value of 9 tons 4 cwts. 3 qrs. 21 lbs. at £14 15*s*. 9*d*. per ton.

When a qr. costs 3*d*. a lb. will cost $\frac{3}{28}d$.; hence, the reason of multiplying the lbs. by $\frac{3}{28}$.

	tons	cwts.	qrs.	lbs.
	9	4	3	21
	1	1	3	$\frac{3}{28}$
	9	4	$11\frac{1}{4}$	
			14	
	£129	9	$1\frac{1}{2}$	
10*s*. = $\frac{1}{2}$ =	4	12	$5\frac{5}{8}$	
5*s*. = $\frac{1}{2}$ =	2	6	$2\frac{13}{16}$	
6*d*. = $\frac{1}{10}$ =		4	$7\frac{77}{160}$	
3*d*. = $\frac{1}{2}$ =		2	$3\frac{237}{320}$	
	£136	14	$9\frac{51}{320}$ (·159375.) *Ans.*	

Admiralty.

39. Find the price of 56 tons 4 cwts. 2 qrs. at £58 7*s*. 6*d*. per ton.

40. „ 38 tons 1 cwt. 16 lbs. at £2 12*s*. 6*d*. per ton.

War Department.

41. Find the price of 56 tons 15 cwts. 2 qrs. 21 lbs. at £24 6*s*. 8*d*. per ton.

115. *For cwts., qrs. and lbs.*

RULE.—Multiply the cwts. by 1, the qrs. by 5, and the lbs. by $2\frac{1}{7}$, and consider the products as in the preceding.

Customs.

Ex. 11. Calculate the duty on 353 cwts. 2 qrs. 7 lbs. of foreign sugar at 17*s.* 4*d.* per cwt.

In this example, and most others, in which the shillings are nearly £1, it is better to find for the difference and subtract. Thus the difference between £1 and 17*s.* 4*d.* is $=2s. 8d.$, and the difference of the values at these two rates will be the answer.

	cwts.	qrs.	lbs	
	353	2	7	@ 17*s.* 4*d.*
	1	5	$2\frac{1}{7}$	
	353	11	3	at £1 per cwt.
$2s. = \frac{1}{10} =$	35	7	$1\frac{1}{3}$	
$8d. = \frac{1}{3} =$	11	15	$8\frac{1}{2}$	
	47	2	10	at 2*s.* 8*d.*
	£306	8	5	*Ans.*

116. *For acres, roods, and perches.*

Rule.—Multiply by 1, 5, $1\frac{1}{2}$, respectively, and consider the products as in the preceding.

Various Departments.

Ex. 12. Find the price of 25 acres 1 rood 10 poles at £42 2*s.* 4*d.* per acre.

When by multiplying by 1, 5, $2\frac{1}{7}$, the value at £1 per acre is obtained, it is easy to find the value at any other rate, as in the annexed work.

	acrs.	rd.	pls.	
	25	1	10	@ £42 2*s.* 4*d.*
	25	1	10	
	1	5	$1\frac{1}{2}$	
	25	6	3	price at £1 per acre
			6	
	151	17	6	price at £6 „
			7	
	1063	2	6	price at £42 „
$2s. = \frac{1}{10}$ of a £ $=$	2	10	$7\frac{1}{2}$	price at 2*s.* „
$4d. = \frac{1}{6}$ of $2s. =$	0	8	$5\frac{1}{4}$	price at 4*d.* „
	£1066	1	$6\frac{3}{4}$	*Ans.*

By Vulgar Fractions.

25 acres 1 rood 10 poles $=25\frac{5}{16}$ acres, and £42 2*s.* 4*d.* $=42\frac{7}{60}$; then $25\frac{5}{16} \times 42\frac{7}{60} = \frac{405 \times 2527}{16 \times 60} = \frac{81 \times 2527}{16 \times 12} = \frac{27 \times 2527}{16 \times 4} = \frac{68229}{64} =$ £1066 1*s.* $6\frac{3}{4}d.$ as before.

By Decimals.

$$
\begin{array}{rr}
 & \text{acrs. rd. pls.} \quad \text{acres} \\
 & 25 \;\; 1 \;\; 20 = 25{\cdot}3125 \\
 & 25{\cdot}3125 \\
 & 42 \\ \hline
 & 506250 \\
 & 1012500 \\ \hline
 & 1063{\cdot}125 \\
2s. = \frac{1}{10} \text{ of a £} = & 2{\cdot}53125 \\
4d. = \frac{1}{6} \text{ of } 2s. = & {\cdot}421875 \\ \hline
 & 1066{\cdot}078125 \\
 & 20 \\ \hline
 & 1{\cdot}562500 \\
 & 12 \\ \hline
 & 6{\cdot}7500 \\
 & 4 \\ \hline
 & 3{\cdot}00
\end{array}
$$

£1066 1*s.* 6¾*d.* *Ans.*

By the Principle in (**31.**)

25 acres 1 rood 10 poles at £42 2*s.* 4*d.* = 101 acres 1 rood at £10 10*s.* 7*d.*; then £10 10*s.* 7*d.* × 100¼ = £1066 1*s.* 6¾*d.* *Ans.*

Various Departments.

42. What is the rent of 245 acres 3 rds. 30 pls. of land at £2 14*s.* per acre?

43. What will be the price of 35 acres 2 rds. 30 pls. at £30 1*s.* 8*d.* per acre?

44. Find the value of 2 rds. 19 pls. 12 yds. of land at £80 13*s.* 4*d.* per acre.

117. Sometimes certain deductions have to be made before the true weight of goods, called the *net* or *neat* weight, is found.

The weight of both goods and box, &c., which contains them is called the *gross* weight.

TARE is the weight of the bag or barrel, which contains the goods, or an *allowance* made for it, usually a per-centage, that is, a deduction per 100 lbs.

Trett is an allowance of 4 lb. in every 104 lb., or $\frac{1}{26}$ part, allowed for dust, &c.

Beamage or **cloff** is an allowance of 2 lb. in every 3 cwt., or $\frac{1}{168}$ part, considered necessary to turn the beam.

Trett and *cloff*, however, are falling into *disuse*.

First tare is deducted ; trett is taken from what remains, and so on. When all deductions are taken we have the *net* weight.

Customs and Excise.

Ex. 13. What is the net weight of the following :

		cwts.	qrs.	lbs.	
J. D.	1	17	2	9	Tare at 18 per cent.
	2	18	2	14	
	3	19	3	0	
	4	20	1	0	
	5	16	3	26	
5 butts		93	0	21	gross.
Tare $\frac{18}{100}=\frac{9}{50}=$		16	3	$2\frac{33}{50}$	
		76	1	$18\frac{17}{50}$	net weight. *Ans.*

In finding the tare we multiply by 9 and divide by 50.

Ex. 14. Raisins.

	Boxes.	cwts.	qrs.	lbs.	
C. L.	5	2	1	24	Tare 6 lbs. each box, what is the net weight?
	5	2	2	1	

	cwts.	qrs.	lbs.	
	2	1	24	
	2	2	1	
	4	3	25	gross.
Tare 10 × 6 = 60 lbs. =		2	4	
	4	1	21	net weight. *Ans.*

45. What is the net weight of

		cwts.	qrs.	lbs.	
F. B.	1	7	2	19	Tare at 17 per cent.
	2	6	3	21	
	3	7	0	14	

XXVII.

Customs.

46. Return for duty 60 chests of liquorice paste, weighing 160 cwts. 2 qrs. gross, allowing for tare 42 lbs. each chest, and for leaves 7 per cent.

118. Flooring, Painting, &c., are frequently calculated by Practice: read Duodecimals again, and work them by taking aliquot parts.

Science and Art, Prisons, Junior Clerks, &c.

Ex. 15. What will the painting of a room cost at 2*s.* 3*d* the square yard, whose height is 10 feet, width 15 feet, and length 19 feet?

$19 \times 10 \times 2 = 380$ the area of the two sides.
$15 \times 10 \times 2 = 300$ the area of the two ends.

680 sq. feet, area of the whole room to be painted, which $\div 9 = 75\frac{5}{8}$ sq. yards. Then $75\frac{5}{8}$ at 2*s.* 3*d* = (multiply the first quantity and divide the 2nd by nine) 680 at 3*d.*

680

3*d.* = $\frac{1}{4}$ of a *s.* = 170*s.* = £8 10*s.* *Ans.*

47. What will the painting of a room cost at 2*s.* 3*d.* the sq. yard, whose height is 15 feet, width 16 feet, and length 18 feet?

Constabulary and Education Offices.

48. Find how many yards of paper $\frac{7}{8}$ of a yard wide will paper a room, whose length is 26 ft. 4 in., breadth 18 ft. 8 in., and its height 12 ft. 3 in.

Various Departments.

49. What is the expense of digging a ditch of which the solid content is 5755 cubic yards at the rate of 15*s.* $7\frac{3}{4}$*d.* per yard?

50. Find the cost of papering a room 16 ft. long, 11 ft. wide, and 10 ft. high, with paper 30 in. broad, at $7\frac{1}{2}$*d.* a yard.

Post Office, Customs, War Department, &c.

51. Find the cost of papering a room, whose height is 12 ft., width 16 ft., and length 20 ft., the price of paper being $2\frac{1}{4}$*d.* the sq. yard.

Miscellaneous Exercises in Practice.

XXVIII.

Post Office.

1. If an officer's pay is 12*s.* 3*d.* per day, what is that per year?

2. A bankrupt owes £2468, and he could pay 15*s.* 6*d.* in the £, what are his effects worth?

3. The charge of carrying the mails by railway to Birmingham is £28 4*s.* 4*d.* per day; how much is it in 365 days?

4. Find the value of 328 articles at 8*s.* 5½*d.* each.
5. „ 1530 „ at 15*s.* 9*d.* each.
6. „ 674 „ at £4 8*s.* 4*d.* each.
7. „ 638 „ at £2 17*s.* 4*d.* each.

Clerks in Post Office, Police Courts, Science and Art, Board of Trade, Fisheries, and Prisons Departments.

8. Find the dividend on £2574 15*s.* at 13*s.* 5*d.* in the pound.

9. Find the cost of making a road, length 9 miles 5 fur. 44 yds., at £25 8*s.* 4*d.* per mile.

10. Find the value of 15 silver plates, each weighing 7 oz. 11 dwts. 6 grs., at 6*s.* 8*d.* per ounce.

11. Find the value of 70 bales of cotton, each weighing 1 cwt. 1 qr. 21 lbs., at £2 3*s.* 1½*d.* per cwt.

12. Find the value of 3 acres 1 rd. 14 pls. at £125 per acre.

13. Find the value of 14 oz. 8 dwts. 20 grs. of gold at £3 17*s.* 6*d.* per ounce.

14. Find the dividend on £3854 10*s* at 8*s.* 2½*d.* in the pound.

15. Find the value of 16 oz. 6 dwts. 20 grs. of gold at £3 17*s.* 6*d.* per ounce.

16 Find the income-tax on £2382 7*s.* 6*d.* at 16*d.* in the pound.

17. Find the value of 983 articles at 3*s.* 10½*d.* each.

18. Find the value of 969 „ at 19*s.* 11½*d.* each.

19. What is the duty on 85960 lbs. of hops at 1*d* $\frac{12}{20}$ of a farthing per lb.?

20. Find the value of 2654¾ at £7 15*s.* 4*d.* each.
21. „ 7394¼ at £12 8*s.* 8¼*d.* each.
22. „ 37 tons 3 cwts. 22 lbs. at £12 17*s.* 7*d.* per cwt.
23. „ 5 tons 3 cwts. 1 qr. 5 lbs. at £3 13*s.* per cwt.

War Department.

24. Find the price of 56 tons 10 cwts. 2 qrs. 21 lbs. at £24 6*s.* 8*d.* per ton.

Surveyor of Taxes.

25. How much income-tax at 10½*d.* in the £ must a man pay who has £1875 per annum?

26. Find (by Practice) the price of 35 bars of gold, each weighing 3 oz. 11 dwts. 9 grs., at £2 12*s.* 6*d.* per oz.

Proposed where Candidates are to know 'Vulgar and Decimal Fractions.'

27. A room is 42 ft. long, 28 ft. broad, and 12 ft. high, what will be

the cost of covering the walls with a paper 2 ft. 3 in. wide, at 9*d.* per yard?

28. Find the cost of 4 cwts. 3 qrs. 22½ lbs. at £1 9*s.* 2*d.* per cwt.

29. If a person's estate is worth £1384 16*s.* per annum, and the rent-charges upon it amount to 14*s.* 9½*d.* in the pound, what is the net annual income?

30. What is the rent of 67 acres of land for 2 years 9 months 11 days, the yearly rent of an acre being £2 5*s.*?

31. A bankrupt whose liabilities are £3236 17*s.* 6*d.* can pay 13*s.* 10*d.* in the pound, what are his assets?

32. Find the value of 6 qrs. 3 bushels 1¾ pecks of wheat at £3 18*s.* 8*d.* per quarter.

33. Find the price of 3 qrs. 20 lbs. 12 oz. at £11 13*s.* 4*d.* per cwt.

34. Find the amount of income-tax on £875 15*s.* at 14*d.* in the £.

35. Find the value of 39 cwts. 1 qr. 20 lbs. at £6 2*s.* 6*d.* per cwt.

36. What is the dividend on £2485 at 17*s.* 9½*d.* in the pound?

37. Find the value of 1001 articles at 8*s.* 7½*d.*

38. Find the value of 5 lbs. 9 oz. 7 dwts. of gold at £3 17*s.* 10*d.* an oz.

39. Find the value of 197 articles at £4 12*s.* 5½*d.*

40. Find the income-tax at 16*d.* in the pound on £1405 15*s.* 11¼*d.*

41. Find the cost of 11 miles 3 fur. and 55 yds. of railway at £32500 per mile.

42. Find the price of 9 tons 11 cwts. 2 qrs. 21 lbs. at £10 11*s.* 4*d.* per cwt.

43. Find the price of 17 acres 3 rds. 15 pls. at £60 3*s.* 4*d.* per acre.

44. Find the value of 6943 sheep, each sheep being worth £1 13*s.* 4½*d.*

45. Find the cost of 7 cwts. 3 qrs. 20½ lbs. at £3 2*s.* 5*d.* per cwt.

46. What is the dividend on £4970 at 8*s.* 10¾*d.* in the pound?

47. Find the value of 3 tons 18 cwts. 3 qrs. 12 lbs. at £2 17*s.* 9*d.* per cwt.

Proposed also in Several Departments.

48. Find the value of 194 articles at £4 12*s.* 5¾*d.* each.

49. Find the value of 3546 pieces of cloth, each piece being worth £1 6*s.* 7¾*d.*

50. What is the price of 329 yards 3 qrs. 2 nls. at 5*s.* 2½*d.* per qr.?

51. Find the value of 821 articles at 13*s.* 10¾*d.*

52. Find the value of 3 lbs. 5 oz. 14 dwts. 12 grs. at 17*s.* 6*d.* per oz.

Education Office, London.

53. Find the price of 8 cwts. 1 qr. 12 lbs. at £4 12*s.* 5½*d.* per quarter.

SIMPLE PROPORTION.

119. RATIO is the relation which exists between any two quantities of the same kind, and is found by dividing the former by the latter: thus, the *ratio* of 6 to 3 is 2, and the ratio of 12 to 4 is 3, for $6 \div 3 = 2$ and $12 \div 4 = 3$.

RATIO is another name for the quotient of *two quantities of the same kind.* Therefore, the quantities may be expressed as the terms of a fraction, the *ratio* being what we have called the real value of the fraction. We may say $\frac{6}{3} = 2$, and $\frac{12}{4} = 3$.

What was formerly called the *terms* of a *fraction*, is now called the *terms* of a *ratio*; what we have called the *numerator* of a fraction, we now call the *antecedent* of a ratio, being usually written before the other term. What we termed the *denominator* of a fraction, we now call the *consequent* of a ratio.

The terms of a ratio are generally written thus, $6 : 3$ (read, 6 to 3) instead of $\frac{6}{3}$, or $6 \div 3$; and $12 : 4$ is the same as $\frac{12}{4}$, or $12 \div 4$.

As has been shown in *Fractions*, the numerator corresponds to the dividend, and the denominator to the divisor; so with the terms of a ratio, the *antecedent* corresponds to the *dividend*, and the *consequent* to the *divisor*; and hence the *quotient* corresponds to, and is, the ratio.

The term *quotient* is used in a more *general* sense than the term ratio, for quotient means, not only the number of times one quantity contains another of the same kind, but is applied to *compound quantities* arising by taking the half, or any other part of a quantity of several denominations. *Ratio* is always expressed by an abstract number, and has reference to the number of times or parts of a time the antecedent contains the consequent. We may say every ratio is a quotient, but we cannot say every quotient is a ratio.

The terms of a ratio may both be *multiplied* or both *divided* by the same number, and the ratio of the results will be the same as that of the original numbers, just as we have seen with respect to the terms of a fraction. For example, 6 : 3 = 18 (6 × 3) : 9 (3 × 3); and 12 : 5 = 60 (12 × 5) : 20 (4 × 5).

120. When ratios are equal, their terms form what is called a PROPORTION, written thus, 6 : 3 : : 18 : 9, and read 6 to 3, as 18 to 9. These four numbers are *proportionals*, that is, 6 bears the same relation to 3, that 18 does to 9; or, that the quotient of 6 divided by 3 is equal to the quotient of 18 divided by 9. Hence, the terms of any two equal fractions will form a proportion, as they imply equal quotients. Then, $\frac{5}{7}=\frac{20}{28}$, becomes, 5 : 7 : : 20 : 28, the numerator of one is, to its denominator, as the numerator of the other, to its denominator.

It follows that any four numbers are proportionals, when the first divided by the second is equal to the third divided by the fourth. They are not proportionals when this is not the case; therefore, by this test, we can determine if any four numbers are proportionals.

Take the four numbers 12, 6, 10, and 4, and we see that $12 \div 6 = 2$, but $10 \div 4 = 2\frac{1}{2}$, therefore these four numbers are not proportionals. Again, 15, 9, 10, and 6; now $15 \div 9 = 1\frac{2}{3}$, and $10 \div 6 = 1\frac{2}{3}$, these four numbers are proportionals, since they have *equal* ratios; and they may be written 15 : 9 : : 10 : 6. The second and third terms of a proportion are called *means* (9 and 10); the first and fourth *extremes* (15 and 6). When four numbers form a proportion, *the product of the means is equal to the product of the extremes.* Thus, 6 : 3 : : 8 : 4; here, 6 × 4, the extremes, = 8 × 3, the means, = 24.

If the product of any two numbers be equal to the product of any other two numbers, the four numbers will form a proportion, the factors of one product forming means, and those of the other forming extremes. This is another test by which we know if four numbers are proportionals.

We may make any change in writing four proportionals, provided the same two terms still form means or extremes, thus:—

We may say,

1°. *The first term : to the second term : : the third term : the fourth term.*

2°. *The first term : the third term : : the second term : the fourth term.*

3°. *The second term : the first term : : the fourth term : the third term.*

4°. *The second term : the fourth term : : the first term : the third term.*

By adding,

5°. *The first term + the second : the second term : : the third term + the fourth : the fourth term.*

In the above explanations, we have considered the terms *abstract* numbers. The first and second terms, when compound quantities are considered, must be of the same kind, and also the third and fourth of the same kind, since we cannot compare quantities of *different* kinds so as to have any *ratio* or relation between them. It has been shown that we cannot express one quantity as a fraction of another quantity of a different kind.

121. From the product of the means being equal to the product of the extremes, the rule for finding a *fourth proportional* to three given numbers may be easily derived. There are always *three terms* given; which must be, either *two extremes* and *one mean*, or *two means* and *one extreme*. If two extremes and one mean be given. From the two extremes, say 10 and 6, we find the product of the extremes, 60; now, as the product of the means is also 60, and we have one mean, say 4, which is *one factor* of 60, considered as product of means; hence, 60÷4=15, the other mean. Therefore we can say, 10 : 15 : : 4 : 6; or, 15 : 10 : : 6 : 4, &c.

It follows that —*product of means ÷ one extreme = the other extreme*; and—*product of extremes÷one mean = the other mean.*

In this rule there are always three numbers given to find a fourth, hence it has been called 'The Rule of Three.' It has also been called 'The Golden Rule,' from its great importance.

122. *To find a fourth proportional to three quantities.*

Rule.—Divide the product of the second and third terms by the first.

Ex. Find a fourth proportional to 8, 6, and 24.

8 : 6 : : 24
6
———
8)144
———
18 *Ans.*

Proof.

By **120.**

8 ÷ 6 = $1\frac{1}{3}$
24 ÷ 18 = $1\frac{1}{3}$

Proof.

The extremes 8 × 18 = 144
The means 6 × 24 = 144

Find fourth proportionals to:

1.* 16 : 18 : : 24.
2. 12 : 30 : : 50.
3. 10 : 8 : : 80.
4. 24 : 100 : : 168.
5. 36 : 12 : : 720.
6. 7 : 14 : : 28.
7. 23 : 69 : : 177.
8. 25 : 125 : : 1025.

123. GENERAL RULE FOR PROPORTION.

1°. Set down the term which corresponds to the answer in the third place.

2°. If, from the nature of the question, the answer is to be *more* than the third term, set down the *greater* of the other two terms in the second place. If the answer is to be

---

* *Answers.*

(1.) 27. (2.) 125. (3.) 64. (4.) 700.
(5.) 240. (6.) 56. (7.) 531. (8.) 5125.

less than the third term, write the *least* of the other two in the second place.

3°. Then divide the product of the second and third terms by the first; the quotient will be the fourth term or answer.

CASE I.

124. When the three given terms are each of one denomination.

Various Departments.

Ex. 1. A coach goes from London to Liverpool at the rate of 9 miles per hour in 24 hours; in what time would the distance be travelled by a train travelling 32 miles per hour?

We see at once that the answer is to be in hours; therefore 24 is written in third place. Anything going 32 miles an hour will take less time than 9 miles an hour; hence 32 will be the first term.

mls. mls. hrs.
32 : 9 :: 24
9
32)216 ($6\frac{3}{4}$ hrs. *Ans.*
192
$\frac{24}{32}=\frac{3}{4}$

By First Principles.

24 hours at rate of 9 miles $=24\times 9=216$, the number of miles travelled; whence $216\div 32=6\frac{3}{4}$ the number of hours necessary to travel the distance at 32 miles per hour.

XXIX.

Post Office (Provincial).

1. How much land at 16*s.* an acre shall be given in exchange for 480 acres at 15*s.* an acre?

2. If a pasture can maintain 7 horses 6 weeks, how long can it maintain 35 horses?

3. If £100 gain £10 in 12 months, how much will gain the same in 9 months?

4. If 100 men can finish a work in 12 days, how many men are sufficient to do the same in 3 days?

Where 'Vulgar and Decimal' Fractions are required.

5. If 5 men can do a piece of work in 27 days, how long would 6 men be in doing double the quantity?

6. If by working 9 hours a day I can finish a piece of work in 12 weeks, how long shall I take to finish it if I work 8 hours per day?

7. A man by working $6\frac{1}{4}$ hours a day does a piece of work in 12 days; how many hours a day must he work in order to do it in $7\frac{1}{2}$ days?

CASE II.

125. *When the first two terms are of different denominations, and the third term of one denomination.*

RULE.—Reduce the first and second terms to the lowest denomination mentioned in either, then proceed as in *Case I.*

Various Departments.

Ex. 2. Find the income-tax on £7980 10*s*. at 7*d*. in the pound.

The reason of the statement of this example is precisely similar to that given in Ex. 1. We reduce, according to the Rule, the first and second terms to shillings, and then multiply the second and third terms together, and divide the product by the first term 20, which gives $55863\frac{1}{2}$ the *ans.* in pence.

£1 : £7980 10*s*. : : 7*d*.
20   20
20   159610
7
2,0)111727,0
12)$55863\frac{10}{20}(=\frac{1}{2})$
2,0)465,5 $3\frac{1}{2}$
£232 15*s*. $3\frac{1}{2}$*d*. *Ans.*

By First Principles.

Income-tax on £7980 10*s*. @ 1*d*. per £ = $7980\frac{1}{2}$*d*.
" " @ 7*d*. " = $7980\frac{1}{2} \times 7 = 55863\frac{1}{2}$*d*.
= £232 15*s*. $3\frac{1}{2}$*d*. *Ans.*

Post Office.

8. As £1648 6*s*. 9*d*. : £505 10*s*. 4*d*. : : £52.

9. If a horse cost 1*s*. 11*d*. per day, what will be the charge of 11 horses for a year?

10. How much is due for 95 days, the salary being £28 7*s*. per annum?

11. What must £2 pay towards a tax when £83 12*s*. 4*d*. is assessed on £650 13*s*. 4*d*.?

War Department, Science and Art, Police Courts, &c.

12. If a train going 25 miles an hour performs a certain distance in $4\frac{1}{2}$ hours, how long would a train going 30 miles an hour take to perform the same?

Various Departments.

13. What is the income-tax on £6150 10*s.* at 7*d.* in the pound?

14. Find the income-tax on £17030 5*s.* at 7*d.* in the pound.

15.* Find the income-tax on £6030 14*s.* 4½*d.* at 16*d.* in the pound.

India Office.

16. If the yearly profits of an investment be £11 9*s.* 6*d.* per cent, how much must be invested in order to produce an annual return of £640 13*s.* 9*d.*?

CASE III.

126. *When the third term is of more denominations than one.*

RULE.—Reduce it to the lowest denomination, and then proceed as before. The answer will be in that denomination to which the third has been reduced.

Note.—If the third term can be conveniently multiplied by the second, it will be unnecessary to reduce it.

India Board.

EX. 3. If a pocket of hops weighing 1 cwt. 3 qrs. 12 lbs. cost £7 13*s.* what is the price per cwt.?

In the annexed operation the first and second terms are reduced to lbs., and the third term to shillings, before multiplying, and the answer is consequently found in shillings.

```
cwts. qrs. lbs.   cwt.
 1    3   12   :   1   ::  £7 13s.
 4                112       20
 —                ———      ———
 7                112      153
28                         112
——                         ————
208                        1836
                          153
                          —————
                    208)17136(82s. 4d.
                        1664  ————————
                        ————      £4 2s. 4 8/13 d. Ans.
                          496
                          416
                          ———
                           80
                           12
                          ———
                          960
                          832
                          ———
                          128/208 = 8/13
```

---

* Proposed to same as No. 12.

By First Principles.

1 cwt. 3 qrs. 12 lbs. = 208 lbs.
∴ Cost of 208 lbs. = £7 13*s.* = 153*s.*
∴ Cost of 1 lb. = $\frac{153}{208}$*s.*
Cost of 112 lbs. $=\frac{153\times112}{208}=\frac{153\times7}{13}=\frac{1071}{13}$*s.*
$=82\frac{5}{13}$*s.* = £4 2*s.* $4\frac{8}{13}$*d.* *Ans.*

Post Office.

17. A bankrupt owes £4968, he has only money sufficient to pay 9*s.* 7*d.* in the £; how much money has he to pay his debts?

18. What is the value of 19½ chaldrons of coals at £1 11*s.* 6*d.* per chaldron?

19. If £70 16*s.* 4*d.* is the cost of 16 cwts. 4 lbs. what would 1 ton 2 cwts. 3 qrs. cost?

20. What is the half year's rent on 547 acres of land at 15*s.* 6*d.* per acre?

Audit Office.

21. If 124 tons 16 cwts. 2 qrs. 16 lbs. cost £1613 19*s.* 6*d.* what will 2 tons cost?

22. If 37 cwts. 3 qrs. 22 lbs. cost £477 16*s.* 8*d.* what will 1 cwt. cost?

Where Candidates are to know 'Arithmetic, including Vulgar and Decimal Fractions.'

23. What weight of sugar may be bought for £93 12*s.* when the cost of 6 cwts. 2 qrs. is £27 14*s.* 8*d.* ?

24. What is the tax on a house rented at £327 12*s.* 6*d.* if the tax on a house rented at 35 guineas is £6 8*s.* 7½*d.* ?

CONTRACTIONS.

127. I. The operation may be often contracted by dividing the first and second, or first and third terms by any number that will *measure* them, and using the quotients instead of these terms themselves.

NOTE.—We can never contract the second and third terms

Ex. 4. What is the price of 17 cwts. 2 qrs. 14 lbs. at £44 16*s.* per cwt.?

After reducing the three terms, we contract the first and third. It will be found most convenient to write the dividends above and the divisors under a line, when the example has been properly stated.

$$\begin{array}{rcrrrcrr}
\text{cwt.} & & \text{cwts.} & \text{qrs.} & \text{lbs.} & & £ & s. \\
1 & : & 17 & 2 & 14 & :: & 44 & 16 \\
\underline{112} & & \underline{4} & & & & \underline{20} & \\
112) & & 70 & & & & 896 & \\
 & & \underline{28} & & & & & \\
 & & 1974 & & & & &
\end{array}$$

$$\frac{1974 \times \cancel{896}}{\cancel{112}} = \frac{1974 \times 8}{1} =$$

15792*s.* = £789 12*s.* *Ans.*

25. The house rent of a labourer for 4 weeks is 9*s.* 4*d.*, what is it for 2 years ?

26. When 10 cows can be bought for £115 3*s.* 9*d.*, how many, at the same rate, will £403 3*s.* 1½*d.* buy ?

27. When 24 cwts. of hay cost £2 15*s.*, how many cwts. can I buy for £8 17*s.* 6*d.* ?

II. 1°. Having stated the example, multiply the *first* and *second* or *first* and *third* terms by any number which will produce products, wanting the *low denominations* of these terms.

2°. Use these products instead of the terms themselves.

Ex. 5. How much linen may be bought for £41 12*s.* 6*d.*, if 405 yards cost £69 7*s.* 6*d.*

Looking at 12*s.* 6*d.* and 7*s.* 6*d.* we see at once that multiplying by 8 will give pounds, without either shillings or pence in the product.

$$\begin{array}{rrrcrrrcr}
£ & s. & d. & & £ & s. & d. & & \text{yds.} \\
69 & 7 & 6 & : & 41 & 12 & 6 & :: & 405 \\
 & & \underline{8} & & & & \underline{8} & & \\
 & 555 & & & & 333 & & &
\end{array}$$

$$\frac{333 \times \cancel{405}}{\cancel{555}} = \frac{\cancel{333} \times 81}{\cancel{111}} = \frac{3 \times 81}{1} =$$

243 yds. *Ans.*

28. What will be the price of 8 cwts. 1 qr. 7 lbs. of cheese, when 3 cwts. 2 qrs. 14 lbs. can be bought for £29 ?

29. How much land can be rented for £768 3*s.* 4*d.* when 4 acres are rented at £6 13*s.* 4*d.* ?

30. When 59 yards can be bought for £7 7*s.* 6*d.* how many can be bought for £10 6*s.* 3*d.* ?

31. When 73 lbs. of beef can be had for £1 19*s.* 6½*d.* what will be the price of 7006 lbs. ?

Science and Art, Prisons, Fisheries, &c.

32. If 16 Napoleons are worth £12 14*s.* 8*d.* what is the value of 105 Napoleons ?

33. If 17 cwts 3 qrs. 14 lbs. of barley cost £8 18*s.* 9*d.*, how much may be bought for £2 16*s.* 3*d.* ?

Various Departments.

34. When the income-tax was 7*d.* in the pound a person paid £24 7*s.* 6*d.*, what additional sum does he pay now that the income-tax is 16*d.* in the pound ?

128. ***If fractions be in some, or in all the terms.***

RULE.—Reduce them to improper fractions, if necessary, and proceed as in the general rule ; that is, invert the terms of the fraction in the first place, and then multiply the three terms of the proportion together.

NOTE.—The first and second terms before being multiplied must be fractions of the *same* denomination.

Various Departments.

Ex. 6. A ship is worth £16000, and a person possessed of $\frac{5}{16}$ of it sells $\frac{3}{8}$ of his share ; what has he remaining, and what is it worth ?

$\frac{3}{8}$ of $\frac{5}{16}=\frac{15}{128}$, what he sold ; then take $\frac{15}{128}$ from $\frac{5}{16}=\frac{40}{128}-\frac{15}{128}=\frac{25}{128}$, what he has remaining.

The whole ship being $\frac{128}{128}$, we have

By decimals.

1 : ·1953125 :: £16000
16000

£3125·0000000 *Ans.*

$\frac{128}{128}$: $\frac{25}{128}$ :: £16000

128 : 25 :: 16000

$$\frac{\cancel{16000} \times 25}{\cancel{128}} = \frac{\cancel{1000} \times 25}{\cancel{8}}$$

$= 125 \times 25 =$ £3125 *Ans.*

Again,

Since he possesses $\frac{25}{128}$, we may find $\frac{1}{128}$ thus : £16000 ÷ 128 = £125 ; hence $\frac{25}{128}$ = £125 × 25 = £3125 *Ans.*

Or, one-sixteenth = £1000 ; hence, $\frac{5}{16}$ = £5000, what he had at first of the ship ; then $\frac{3}{8}$ of £5000 = £1875, what he sold ; whence £5000 − £1875 = £3125 *Ans.* as before.

Various Departments.

35. If $\frac{3}{5}$ of a lb. cost 8*s.* 2*d.* what must I pay for $5\frac{1}{7}$ lbs. ?

36. If $\frac{11}{15}$ of a cwt. be worth £4 19*s.* what will be the price of $\frac{5}{9}$ of a ton ?

37. If $\frac{13}{15}$ of a cwt. be worth £7 3*s.* what will be the value of $\frac{6}{11}$ of a ton ?

38. If $2\frac{3}{4}$ lbs. of tea cost 12*s.* 9*d.* what will $\frac{3}{16}$ of a pound cost ? (Solve by decimals.)

39. If $2\frac{3}{8}$ lbs. of tea cost 9*s.* 6*d.* what will $\frac{5}{12}$ of a pound cost ?

129. *To divide any quantity or number into parts, proportional to the parts of another quantity or number.*

Ex. 7. Three labourers earn £2 13*s.* 4*d.*, one working 8 days, the second 5, and the third 3 days ; how much is each to receive ? *

Here 8 + 5 + 3 = 16, the total number of days worked, and £2 13*s.* 4*d.* ÷ 16 = 3*s.* 4*d.* = the value of a day's work ; hence,

3*s.* 4*d.* × 8 = £1 6*s.* 8*d.* the first man's share.
3*s.* 4*d.* × 5 = 16*s.* 8*d.* the second „
3*s.* 4*d.* × 3 = 10*s* 0*d.* the third „

Ex. 8. Suppose the three labourers to work the same number of days, and to earn £4 17*s.* 6*d.* ; but the first only works $4\frac{1}{2}$ hours per day, the second 6 hours, and the third 8 hours, how will the money be divided ? †

$$\left.\begin{array}{l} 8\times 4\frac{1}{2}=36 \\ 5\times 6\ \ =30 \\ 3\times 8\ \ =24 \end{array}\right\}$$ 90 total number of hours worked by all ; then

£4 17 6*d.* ÷ 90 = 1*s.* 1*d.* the wages per hour.
36 × 1*s.* 1*d.* = £1 19*s.* first man's share.
30 × 1*s.* 1*d.* = £1 12*s.* 6*d.* second „
24 × 1*s.* 1*d.* = £1 6*s.* third „

40. There is a certain work completed by three men for £2 17*s.* 6*d.* ; one man worked 50 hours, a second, 40 hours, and a third, 10 hours, how much should each receive ?

41. Three farmers hold 30, 40, and 50 acres, respectively ; the tax on the whole is £77 16*s.* 8*d.* How much has each man to pay ?

42. A bankrupt owes one man £800, and another, £900 ; his effects are worth £272 : how much should each creditor receive ?

43. Two men bought an ox weighing 5 cwts. 2 qrs. 20 lbs., and sold it at $7\frac{1}{2}$*d.* per lb., what is each man's share of the selling price ; one gave half-a-crown for every 2*s.* of the other in the purchase money ?

---

* By statement 8+5+3=16 the total number of days : then—
16 : 8 :: £2 13 4 : £1 6 8 1st man's.
16 : 5 :: £2 13 4 : £0 16 8 2nd „
16 : 3 :: £2 13 4 : £0 10 0 3rd „

† By statement—
90 : 36 :: £4 17 6 : £1 19 0 1st man's.
90 : 30 :: £4 17 6 : £1 12 6 2nd „
90 : 24 :: £4 17 6 : £1 6 0 3rd „

COMPOUND PROPORTION.

130. When *more* than two ratios can be formed; or, when an example in *Simple Proportion* is modified by one or more conditions, it is *Compound Proportion.*

Various Departments.

Ex. 1. If 21 men take 8 days to dig 20 acres, how many acres will 16 men dig in 12 days?

Had this question been thus expressed: If 21 men dig 20 acres, how many acres will 16 men dig? it would have been worked thus:—

acres
As 21 : 16 :: 20 : $15\frac{5}{21}$ acres.

But the 16 men are to work 12 days, whilst the 21 men are only to work 8 days; hence we have,

days days ac. ac. r. p.
8 : 12 :: $15\frac{5}{21}$: 22 3 $17\frac{1}{7}$ *Ans.*

Instead of giving the two statements in full, as above, it is much preferable to *abridge* them in the following manner:—

Rule.—1°. Set the term which is the same name as the answer in the third place.

2°. Compare the other terms by pairs, and consider what effect each ratio will have on the answer, and set them down according to the General Rule for Simple Proportion.

3°. Then multiply all the second terms together, and multiply this product by the third term.

4°. Divide this last product by the product of all the first terms, for the answer.

For Contractions.—Divide *any* term in the *first* place and any in the *second* place by common factors; or, any of the *first* terms and any of the *third* terms by common factors.

Having stated the same question according to the rule, we multiply 21 and 8, the two first terms, and 16 and 12, the two second terms, and then proceed as in Simple Proportion.

$$\begin{array}{rcrcr} & & & & \text{acres.}\\ 21 & : & 16 & :: & 20\\ 8 & : & 12 & :: & \\ \hline 168 & & 192 & & 20\\ & & & & 192\\ \hline & & & 168) & 3840\\ \hline \end{array}$$

22 ac. 3 rds. 17$\frac{1}{7}$ pls. *Ans.*

Contracted.

The dividends are written above, and the divisors below, a line; then they are contracted by dividing by common factors.

$$\begin{array}{rcrcr} 21 & : & 16 & :: & 20\\ 8 & : & 12 & & \\ \hline \end{array}$$

$$\frac{\cancel{16} \times \cancel{12} \times 20}{\cancel{21} \times \cancel{8}} = \frac{2 \times 4 \times 20}{7}$$

$= \frac{160}{7} = 22$ ac. 3 r. 17$\frac{1}{7}$ p.

By First Principles.

21 men in 8 days dig 20 acres

1 man in 8 days will dig $\frac{20}{21}$ acre

1 man in 1 day will dig $\frac{20}{168}$ acre

whence 16 men in 1 day will dig $\frac{20 \times 16}{168} = \frac{20 \times 2}{21}$

„ 16 „ 12 days „ $\frac{\cancel{12} \times 20 \times 2}{\cancel{21}} = \frac{8 \times 20}{7} = \frac{160}{7} = 22$ acres 3 rds. 17$\frac{1}{7}$ pls. *Ans.*

Education and Constabulary Offices.

Ex. 2. If 20 horses and 196 sheep can be kept 18 days for £151 10*s.*, what sum will keep 15 horses and 72 sheep for 8 days, supposing 5 horses to eat as much as 76 sheep?

When 5 horses = 76 sheep, 20 horses = 304 sheep; therefore 20 horses and 196 sheep = 304 + 196 = 500 sheep. Again, 15 horses = 3 × 76 = 228 sheep, and 15 horses and 72 sheep = 300 sheep. Hence the statement.

By First Principles.	£	*s.*	*d.*
500 sheep for 18 days =	151	10	
100 sheep for 18 days =	30	6	
100 sheep for 1 day =	1	13	8
300 sheep for 1 day =	5	1	
300 sheep for 8 days =	£40	8*s.* *Ans.*	

$$\begin{array}{rcrcrr} & & & & \pounds & s.\\ 5\cancel{0}\cancel{0} & : & 3\cancel{0}\cancel{0} & :: & 151 & 10 :\\ 18 & : & 8 & & & 4\\ \hline \cancel{9}\cancel{0} & : & \cancel{2}\cancel{4} & & & \\ 15 & : & 4 & & 15)606 & \\ \hline & & & & \pounds 40 & 8s. \end{array}$$

XXX.

Post Office, War Department, Prisons, &c.

1. If the carriage of 21 cwts. for 4 miles costs 11*s.* 8*d.*, find the cost of carrying 7 cwts. 47 miles ?

2. If 7 men can mow 34 acres in 12 days of $8\frac{1}{4}$ hours each, how many can be mown by 30 men in 11 days of $7\frac{4}{5}$ hours each?

Post Office (Provincial Clerks).

3. If £600 gain £45 in 18 months how much will £100 gain in a year?

Customs.

4. If £100 in 12 months gain £7 interest, what is the interest of £571 for 6 years?

Various Departments.

5. If 12 men working 8 hours a day can build a house in 70 days, in how many days might it be built by 42 men working 10 hours a day?

6. If 6 iron bars 4 ft. long, 3 inches broad, and 2 inches thick, weigh 288 lbs., how much will 15 weigh, each $6\frac{1}{2}$ ft. long, 4 in. broad, and 3 in. thick?

7. If 5 men receive £18 15*s.* wages for 12 weeks, what will be the wages of 16 men for 20 weeks?

8. If 49 men can do a piece of work in 130 days of 8 hours each, how many hours a day must 196 men work to do as much in 26 days?

9. If 780 men on full rations consume 91 quarters of wheat in 49 days, how many on half rations will consume 69 quarters in 9 days?

10. If 3 men can mow 7 acres of grass in 5 days of 9 hours each, in how many days of 10 hours each will 5 men mow 35 acres?

11. A coach goes 9 miles in an hour, and a railway train goes 23 miles while the coach goes 11. How much time will be saved in a journey of 138 miles by taking the train instead of the coach?

12. If the carriage of 65 lbs. for 120 miles cost 6*s.*, how much may I have carried 750 miles for a guinea?

XXXI.

MISCELLANEOUS EXERCISES IN PROPORTION.

Audit Office.

1. If 3 cwts. cost £11 1*s.* 3*d.* what will be the price of 16 cwts. 1 qr. 21 lbs.?

2. If 3 cwts. 13 lbs. cost £14 2*s.* 9*d.* what will 7 cwts. 2 qrs. 25 lbs. cost?

3. If 3875 cwts. cost £34596 9*s.* $8\frac{1}{4}$*d.* what will 1 cwt. cost?

4. If 124 tons 16 cwts. 2 qrs. 16 lbs. cost £1613 19*s.* 6*d.* what will 2 tons cost?

Admiralty.

5. If 84 men can do a piece of work in 2 months, how long will it take 12 men to do the same?

6. If the carriage of 18 tons 13 cwts. 3 qrs. 14 lbs. cost 18*s.* $7\frac{3}{4}$*d.* what will be the cost of the carriage of 256 tons 19 cwts. 2 qrs. 26 lbs.?

Inland Revenue.

7. If a penny loaf weighs 4 oz. when flour is at 4*s.* the peck, what should it weigh when flour is 5*s.* 4*d.* the peck?

8. Assuming that 32 bricks will make 9 square feet of pavement, how many will be required to pave a yard 63 feet long by 36 feet wide?

War Department.

9. What would be the cost of 5000 lbs. of salt beef at £2 15*s.* 4*d.* per cwt.?

10. What would be the charge for fuel for 800 men for 12 months, provided the ration for each man per diem consisted of $1\frac{1}{5}$ foot of wood from 1st May to 31st October, and half that quantity for the remainder of the year, the price of fuel being $6\frac{5}{9}$*d.* per foot?

11. What sum of money will be required per week of 7 days to pay 8 companies, each consisting of 95 men, at 1*s.* $1\frac{1}{2}$*d.* a day for each man?

12. What is the cost of 3 cwts. 3 qrs. 3 lbs. of beef at $4\frac{15}{16}$*d.* per lb.?

13. If gunpowder cost £4 18*s.* 6*d.* per cwt. what is the value of 14 cwts. 2 qrs. 7 lbs.?

14. What ought a loaf to weigh when wheat is at 4*s.* a bushel, if when at 5*s.* 3*d.* a bushel it weighs 2 lbs. 8 oz.?

15. If the carriage of 8 cwts. of coals 128 miles cost 48*s.* what would be the charge for 4 cwts. 32 miles at the same rate?

Post Office.

16. What is the income of a person who pays £22 7*s.* 5*d.* for income-tax at the rate of 7*d.* in the pound?

17. What must an income of £400 pay towards a tax at 4*s.* 6*d* in the £?

18. $44\frac{1}{2}$ guineas used to be coined out of 1 pound standard gold, how many sovereigns are now coined out of this weight?

19. If a bankrupt is able to pay 12*s.* $6\frac{1}{2}$*d.* in the £ and his debts amount to £3600, what is his estate worth?

20. If a bankrupt, whose effects are worth £1181 12*s.* 6*d.*, owes £4726 10*s.*, how much will he be able to pay in the £?

Customs, Junior Clerks, War Department, Post Office, Department of Science and Art, &c.

21. If a 4*d.* loaf weighs 24 oz. when wheat is at 50*s.* per quarter, what is the price of wheat when the 6*d.* loaf weighs 30 oz.?

22. If it cost £18 5*s.* 9*d.* to carpet a room 22 ft. long and 19 ft. wide, how much will a carpet of the same kind cost for a room 27 ft. long and 16 ft. wide?

23. Find the cost of carpeting a room that is 25 ft. long by 20 ft. wide, the price of a yard of carpet 2 ft. 6 in. wide being 4*s.* 3*d.*?

24. A person who values his property at £3500 insures half of it at 5*s.* 6*d.* per cent., and half at 4*s.* 6*d.* per cent; what does it cost him?

25. Reduce £125 10*s.* into francs and centimes, exchanged at 23 francs 6 centimes per £ (100 centimes = 1 franc).

26. If 17 cwts. 3 qrs. 14 lbs. of barley cost £8 18*s.* 9*d.*, how much may be bought for £5 12*s.* 6*d.*?

27. How much will a creditor lose on a debt of £5342 5*s.* when a bankrupt can only pay 13*s.* 6*d.* in the pound?

28. Find the income-tax on £7530 14*s.* 4½ at 16*d.* in the pound.

29. If 12 men can mow a field of grass in 5 days, in how many days could 8 men mow the same?

30. How many yards of carpet, 2 ft. 6 in. broad, will cover a room which is 32 ft. long and 15 ft. broad?

31. If 5 gallons of oil cost 18*s.* 4*d.* find the cost of 13 gals. 3 qrts. 1 pint.

32. What is the income corresponding to an income-tax of £108 1*s.* 4½*d.* at the rate of 9*d.* in the pound?

Customs and Excise.

33. Required the cost of 17 hogsheads of tobacco, each weighing 11 cwts. 2 qrs. 19 lbs. at £19 8*s.* per cwt.

War Department.

34. If the price of 1 oz. of gold is £3 10*s.* what is the price of 14 ingots, each weighing 3 lbs. 7 oz. 14 dwts. 21 grs.?

35. If 1 yard of cloth cost 15*s.* 6¾*d.* what will be the price of 32½ yards at the same rate?

Various Departments.

36. If 780 men on full rations consume 221 quarters of wheat in 119 days, how many on half rations will consume 437 quarters in 57 days?

37. If the shares of a railway are at 76½ when consols are at 93½, what should be their price when consols are at 89¼?

38. How many yards of matting 7·3 ft. broad will cover a floor 27·3 ft. long and 10·083 ft. broad?

39.* If $3\frac{2}{5}$ lbs. of tea cost 15*s.* 3*d.* how many pounds can I buy for £4 3*s.* $10\frac{1}{2}$*d.*

40. A tobacconist mixed together 80 lbs. of tobacco at 14*d.* per lb., 100 lbs. at 20*d.*, 60 lbs. at 4*s.* 10*d.*, and 20 lbs. at 2*s.* 10*d.* per lb.; what will be the value of 3 oz. of this mixture?

41. Find the income-tax on £8313 5*s.* at 7*d.* in the £.

42. If a bar of gold weighing 7 lbs. 1 oz. 14 dwts. is worth £257 2*s.* what is that per oz.?

43. If 12 men can dig a trench 15 yds. long and 4 broad in 3 days of 12 hours each, in how many days of 9 hours each can 8 men dig a trench 20 yards long and 8 broad?

44. If £59 10*s.* will buy 25 cwts. 2 qrs. of sugar, what quantity will £6 14*s.* 2*d.* buy?

45. If a penny loaf weighs 5 oz. when wheat is at 62 shillings the quarter, what ought it to weigh when wheat is 64*s.* the quarter?

46. If by paying down £70 17*s.* 6*d.* a person becomes entitled to £2 9*s.* 7*d.* a year, what income should he gain by paying £2151 17*s.* 8*d.* in the same way?

47. If a tax on an income of £1132 amount to £125 1*s.* $5\frac{7}{8}$*d.* what is that in the pound?

48. If the carriage of 107 cwts. 10 lbs. cost £37 9*s.* $7\frac{1}{2}$*d.* what would the carriage of a ton cost?

49. The shares of a given railway are at $59\frac{1}{2}$ when consols are at $93\frac{1}{2}$, what should be their value when consols are at $91\frac{1}{2}$?

50. If the price of 3 bushels of wheat is 16*s.* 9*d.* find the price of 12 quarters 2 bushels 1 peck.

51. Find the income corresponding to an income-tax of £50 7*s.* 1*d.* at 5*d.* in the pound.

52. Gold of the value of £423284 arrives from Australia; what is its weight, the price being £3 18*s.* per oz.?

53. Find the value of 14 cwts. 3 qrs. 17 lbs of sugar at £2 17*s.* per cwt.

54. If $3\frac{1}{2}$ lbs. of sugar cost 2*s.* what will be the price of 54 cwts. 2 qrs. 7 lbs.?

---

* 15*s.* 3*d.* $=15\frac{1}{4}=15\cdot25$, and $3\frac{2}{5}=3\cdot4$, and £4 3*s.* $10\frac{1}{2}$*d.* $=83\frac{21}{24}\ (\frac{7}{8})$*s.* $=$ 83·875 : then

$\cancel{1}\cancel{5}\cdot\cancel{2}\cancel{5} : \cancel{8}\cancel{3}\cdot\cancel{8}\cancel{7}\cancel{5} :: 3\cdot4$ lbs.
$\cancel{3}\cdot\cancel{0}5 : \cancel{1}\cancel{6}\cdot\cancel{7}\cancel{7}5 ::$
$\cdot\cancel{6}\cancel{1} : \cancel{3}\cdot\cancel{3}\cancel{5}\cancel{5} ::$

·61 will divide 1st and 2nd, and we have 1 : 5·5. Then 5·5 × 3·4 =18·7 lbs. *Ans.*

55. Find the amount of income-tax on £945 17*s.* 6*d.* at 14*d.* in the £.

56.* A person fails for £1078 and his effects are worth £370 18*s.* $5\frac{1}{2}$*d.* what will be the dividend on £78 10*s.* ?

57.† If 6 cwts. cost £9 6*s.* what is the value of 18 cwts. 1 qr. 21 lbs. ?

58.† Find what is the income of a person who pays an income-tax of £77 1*s.* $3\frac{3}{4}$*d.* when the rate is 10*d.* in the pound.

Education Office, Dublin.

59. If 48 pioneers, in 5 days of $12\frac{1}{2}$ hours long, can dig a trench 139·75 yards long, $4\frac{1}{2}$ yards wide and $2\frac{1}{2}$ yards deep, how many hours per day must 90 pioneers work during 42 days, in order to dig a trench $4910\frac{1}{16}$ yards long, $4\frac{7}{8}$ yards wide, and $3\frac{1}{5}$ yards deep ?

THE CHAIN RULE.

CONJOINED PROPORTION OR CHAIN RULE.

131. Conjoined Proportion or Chain Rule is the combination of several ratios or proportions, through which the ratio of the first and last term is discovered.

This rule relates principally to the exchanges between nations, but is frequently applied to other mercantile transactions.

132. *To find how much of the quantity mentioned first, is equivalent to a certain portion of the quantity mentioned last.*

RULE.—1°. Call the first terms in each part of the general question antecedents, and the following ones consequents.

2°. Then place the antecedents in a vertical column, and the consequents in another column to the right of the former: making the second antecedent the same name as the first consequent, the third antecedent the same name as the second consequent, and so on.

---

* Proposed in Post Office.

† Proposed in Admiralty.

3°. Write the odd term under the first column.

4°. Then for the answer divide the product of the antecedents by the product of the consequents.

133. *To find how much of the thing mentioned last, is equivalent to a certain quantity of the thing mentioned first.*

Rule.—1°. Arrange the terms as before, but place the odd term under the column of consequents.

2°. Multiply and divide as in the previous rule.

Ex. 1. If 10 lbs. of flour be worth 30 bushels of wheat, and 20 bushels of wheat worth 10 yds. of cloth, and 60 yds. of cloth worth 120 gals. of brandy, and 30 gals. of brandy worth 60 gals. of cider; how many barrels of flour will purchase 12 gals. of cider?

Antecedents.		Consequents.
10 brls. flour	=	30 bushels wheat.
20 bush. wheat	=	10 yds. cloth.
60 yds. cloth	=	120 gals. brandy.
30 gals. brandy	=	60 gals. cider.
12 gals. cider	=	? brls. flour.

$$\frac{\cancel{10} \times 2\cancel{0} \times \cancel{60} \times \cancel{30} \times 12}{\cancel{30} \times \cancel{10} \times 12\cancel{0} \times \cancel{60}} = \frac{\cancel{12} \times 2}{\cancel{12}} = 2 \text{ brls. of flour. } \textit{Ans.}$$

Had the question been how many gallons of cider will purchase 20 barrels of flour—

Then the work would be

Antecedents.		Consequents.
10	=	30
20	=	10
60	=	120
30	=	60
?	=	20

$$\frac{\cancel{30} \times \cancel{10} \times 120 \times \cancel{60} \times \cancel{20}}{\cancel{10} \times \cancel{20} \times \cancel{60} \times \cancel{30}.} = 120 \text{ gals. cider. } \textit{Ans.}$$

XXXII.

1. If 10 gals. malt spirits be worth 6 gals. of brandy, and 8 gals. of brandy worth 12 gals. of sherry, and 18 gals. of sherry worth 14 of

claret, find how many gallons of malt spirits 21 gallons of claret are worth.

2. If 14 lbs. of butter are worth 10 stones of oats, and 5 sts. of oats =3½ sts. of flour, and 2 sts. of flour = 10 sts. of salt, and 15 sts. of salt = 6 doz. of eggs, how many doz. of eggs are 7 lbs. of butter worth ?

3. If 7 lbs. of beef be worth 8 lbs. of pork, and 9 lbs. of pork = 7 lbs. of bacon, and 20 lbs. of bacon = 18 lbs. mutton, and 12 lbs. mutton = 10 lbs. cheese; how many lbs. of cheese are equivalent to 8 lbs. beef ?

PARTNERSHIP.

134. When several persons join in trade the profits or losses will be proportional to the capital subscribed by each and the time it was employed in business.

Partnership is usually divided into two cases: 1. When the stocks have been in trade the *same* time. 2. When the stocks have been in trade for *different* periods of time.

Ex. 1. Two persons enter into partnership, one puts in £450, the other £330; how are they to divide their first year's profits, which amount to £487 10*s.*?

The sum of the stocks £450 + £330 = £780; then, we have £780 : 450 :: £487 10*s.* : £281 5*s.* the first partner's share.

Again, £780 : £330 :: £487 10*s.* : £206 5*s.* the share of the other partner.

By First Principles.

$$\text{Gain on } \pounds780 = \pounds487\ 10s.\text{, and hence gain on } \pounds1 = \frac{\pounds487\ 10s.}{780}$$

$$\therefore \text{gain on } \pounds450 = \frac{\pounds487\ \ 10s. \times \cancel{450}}{\cancel{780}} = \frac{\pounds487\ \ 10s. \times 15}{26} = \pounds281\ \ 5s.$$

$$\text{and gain on } \pounds330 = \frac{\pounds487\ \ 10s. \times \cancel{330}}{\cancel{780}} = \frac{\pounds487\ \ 10s. \times 11}{26} = \pounds206\ \ 5s.$$

India Board.

Ex. 2. A tradesman starts with a capital of £960, and after 3 years takes another into partnership with £2100. After 4 years more the whole profits amount to £2304; how ought this to be divided between them ?

First man's capital × its time in trade = £960 × 7 = £6720
Second man's capital × its time in trade = £2100 × 4 = £8400
15120

Whence, we have 15120 : 6720 :: £2304 : £1024 share of first; and 15120 : 8400 :: 2304 : £1280 share of second. The reason for the above operation is obvious, for £6720 for one year is the same as £960 for 7 years, so far as gain or loss is concerned.

XXXIII.

1. If I commence business with a capital of £2000, and at the end of 2 years take a partner who has £6000. At the end of 2 years more, how much should my partner receive, our profits being £1000?

2. A, B and C enter into partnership: A puts in £450 for 4 months: B £360 for 6 months: and C £260 for 9 months. What should be each of their shares in a gain of £210?

3. Four persons hold a pasture farm in common, for which they are to pay £80 17*s.*, and into which A puts 8 cows for 3 months; B, 9 cows for 4 months; C, 16 cows for 3 months; and D, 13 cows for 3 months. How much of the rent should each pay?

SIMPLE INTEREST.

135. INTEREST is the price paid for the use of money.

The money borrowed is called the PRINCIPAL.

The sum given for the use of £100 for a year is called the *rate per cent.*

The PRINCIPAL and *interest,* when added together, are called the AMOUNT, that is, the whole amount of money due by the borrower.

If I lend my friend £100 for a year, and if he pay me £5 for the use of it; the £100 lent to him is the *principal*; the £5 which he pays me is the *rate*; and the whole money due at the end of the year, £100 + £5 = £105, is the *amount.*

If the interest be *added* to the principal at the end of certain periods, and this amount considered as *new* principal, *it is called Compound Interest.*

When the interest is *not* so added, it is Simple Interest.

Almost all the examples in finding the interest of money will be understood from the following *proportional* consideration:—

If £100 (cent.) produce a certain sum (the rate) in one year, what will any other sum of money (the given principal) produce, in any other time (the given time)?

136. TO FIND THE INTEREST.—GENERAL RULE.

Multiply the principal by the rate, and that product by the time in years, months or days, and divide by 100, 1200, or 36500, respectively, for the answer.

FOR YEARS.

War Department.

Ex. 1. What is the simple interest on £675 18*s*. for 5 years at $3\frac{1}{2}$ per cent.?

In the annexed example we first multiply by $3\frac{1}{2}$, the rate, then this product by 5, the time in years. We divide this last product by 100, as in (**57.**) and find the answer, £118 5*s*. $7\frac{4}{5}$*d*.

$$\begin{array}{rrr}
 & \pounds 675 & 18 \\
 & & 3\frac{1}{2} \\
\hline
 & 2027 & 14 \\
 & 337 & 19 \\
\hline
 & 2365 & 13 \\
 & & 5 \\
\hline
1{,}00) & 118{,}28 & 5 \\
 & 20 & \\
\hline
 & 5{,}65 & \\
 & 12 & \\
\hline
 & 7{,}80 & \\
 & \overline{100} & =\frac{4}{5}
\end{array}$$

£118 5*s*. $7\frac{4}{5}$*d*. *Ans*.

By First Principles.

Interest of £100 for a year (the rate) = £3 10*s*.

Interest of £1 „ $= \frac{\pounds 3\ 10s.}{100} = \frac{7\cancel{0}s.}{10\cancel{0}} = 8\frac{2}{5}d.$

Interest of £1 for 5 years $= 8\frac{2}{5}d. \times 5 = 3s.\ 6d.$

Whence interest of £675 = 3*s.* 6*d.* × 675 = £118 2*s.* 6*d.* Now since int. of £1 = 3*s.* 6*d.*, int. of 18*s.* = 3*s.* 6*d.* × $\frac{18}{20}$ = 3*s.* 6*d.* × $\frac{9}{10}$ = 3*s.* $1\frac{4}{5}$*d.*; then £118 2*s.* 6*d.* + 3*s.* $1\frac{4}{5}$*d.* = £118 5*s.* $7\frac{4}{5}$*d.* *Ans.*

Questions in this rule may be more conveniently written

$$\frac{£675\ 18s. \times 3\frac{1}{2} \times 5}{100} = \frac{£675\ 18s. \times 7 \times \cancel{5}}{\cancel{2}\cancel{0}\cancel{0}} = \frac{£675\ 18s. \times 7}{40} = \frac{£4731\ 6s.}{40} =$$

£118 5*s.* $7\frac{4}{5}$*d.*

The reason for the operation will be at once evident by considering the question as one in COMPOUND PROPORTION.

FOR MONTHS.

Various Departments.

Ex. 2. Find the interest on £4000 for 9 months at 5 per cent., less income-tax at 14*d.* in the £.

$$\begin{array}{r}
£4000 \\
5 \\
\hline
20000 \\
9 \\
\hline
12\cancel{0}\cancel{0})1800\cancel{0}\cancel{0} \\
\hline
£150
\end{array}\quad \text{the interest.}$$

Income-tax 150 × 14*d.* =

£8 15*s.* tax for a year, and for 9 months = £6 11 3 ($\frac{3}{4}$ of £8 15*s.*)

£143 8 9 net interest

By First Principles.

Interest of £100 for 1 year = £5

" of £100 for 9 months = ($\frac{3}{4}$) = £3 15*s.*

Income-tax of £3 15*s.* for a year at 14*d.* = 4*s.* $4\frac{1}{2}$*d.*; therefore for 9 months = 3*s.* $3\frac{3}{8}$*d.* Whence net int. of £100 for 9 months = £3 15*s.* − 3*s.* $3\frac{3}{8}$*d.* = £3 11*s.* $8\frac{5}{8}$*d.*; hence int. of £4000 = £3 11*s.* $8\frac{5}{8}$*d.* × 40 = £143 8*s.* 9*d.* *Ans.*

FOR DAYS.

Various Departments.

Ex. 3. Find the interest on £1158 17*s.* 6*d.* for 1 year 115 days at $2\frac{1}{2}$ per cent.

We first multiply the principal by 2½, and then this product by 480, the number of days (365 + 115) = 480 = 10 × 8 × 6. We divide the last product by 36500 according to the rule.

$$\begin{array}{rrrrl}
 & \pounds & s. & d. & \\
 & 1158 & 17 & 6 & \\
 & & & 2\frac{1}{2} & \\ \hline
 & 2317 & 15 & 0 & \\
 & 579 & 8 & 9 & \\ \hline
 & 2897 & 3 & 9 & \\
 & & & 10 & \\ \hline
 & 28971 & 17 & 6 & \text{prod. by } 10 \\
 & & & 8 & \\ \hline
 & 231775 & 0 & 0 & \text{prod. by } 80 \\
 & & & 6 & \\ \hline
3650\cancel{0}) & 139065\cancel{0} & 0 & 0 & \text{prod. by } 480
\end{array}$$

$$\begin{array}{rl}
10950 & \\ \hline
29565 & (\pounds 38\ 2s.\ \mathit{Ans.} \\
29200 & \\ \hline
365 & \\
20 & \\ \hline
7300 & \\
7300 & \\ \hline
\end{array}$$

The work of the above more briefly.

$$\pounds 1158\ 17s.\ 6d. = 1158\tfrac{7}{8} : \text{ then } \frac{1158\frac{7}{8} \times 2\frac{1}{2} \times 480}{365 \times 100} = \frac{9271 \times \cancel{5} \times 48\cancel{0}}{8 \times 2 \times 365 \times \cancel{1}\cancel{0}\cancel{0}} =$$

$$\frac{9271 \times \cancel{4}\cancel{8}}{\cancel{8} \times \cancel{2} \times 365 \times 2} = \frac{9271 \times 3}{365 \times 2} = \frac{27813}{730} = \pounds 38\ 2s.\ \mathit{Ans.}$$

By Decimals.

In multiplying by 2·5 we use the rule (**36**).

$$\begin{array}{rr}
\pounds 1158\frac{7}{8} = & 1158{\cdot}875 \\
2\frac{1}{2} = & 2\ 5 \\ \hline
 & 2897{\cdot}187\dot{5} \\
 & 480 \\ \hline
 & 2317750000 \\
 & 115887500 \\ \hline
36500) & 1390650{\cdot}0000 \\ \hline
 & \pounds 38\ 2s.\ \mathit{Ans.}
\end{array}$$

By First Principles.

Interest of £100 for 1 year = 2½ = £2 10s.

hence int. of £1 for 1 year $= \frac{2\frac{1}{2}}{100} = \frac{5}{200} = \frac{1}{40}$

" int. of £1 for 1 day $= \frac{1}{40 \times 365}$

" int. of £1 for 480 days $= \frac{480}{40 \times 365} = \frac{12}{365}$

Whence int. of £1158 17*s.* 6*d.* for 480 days $= \frac{£1158\ 17\ 6 \times 12}{365} = \frac{£13906\ 10}{365} = £38\ 2s.$ *Ans.*

Ex. 4. Find the interest and amount of £225 10*s.* from 23rd September to 11th January following, at 5 per cent.

	days.
September	7
October	31
November	30
December	31
January	11
Total no. of days	110

Then $\frac{£225\frac{1}{2} \times 110 \times \cancel{5}}{\cancel{36500}} = \frac{451 \times 11\cancel{0}}{730\cancel{0} \times 2} = \frac{4961}{1460} =$ £3 7*s.* $11\frac{37}{73}$*d.* *interest.*

$£225\ 10 + £3\ 7\ 11\frac{37}{73} = £228\ 17\ 11\frac{37}{73}$. *amount.*

XXXIV.

Constabulary Office.

1. Find the simple interest on £507 10*s.* for 3¼ years at 4 per cent.

Customs, Post Office, &c.

2. Find the interest on £1257 15*s.* for 5¾ years at 4 per cent. per annum.

Various Departments.

3. Find the interest on £3000 for 8 months, at 4 per cent., less income-tax at 14*d.* in the pound.

4. Find the interest on £260 10*s.* for 3⅓ years at 4¼ per cent.

5. Find the simple interest on £4164 10*s.* for 6 years at 2¾ per cent. per annum.

6. Find the interest on £1250 in 7½ years at 3¼ per cent.

7. Find the interest on £1257 15*s.* for 5¾ years, at 4¼ per cent. per annum.

8. Find the interest on £194 from 3rd March to the 18th December following at 5 per cent. per annum.

9. Find the interest on £260 10*s.* for 3⅓ years at 4¼ per cent.

10. Find the amount of £322 13*s.* for 10¼ years at 3½ per cent. simple interest.

11. Find the interest at 4¾ per cent. on £924 12*s.* 8*d.* lent on 18th March and repaid on the 11th August, the former day being excluded and the latter included.

137. *The following class of questions has been frequently proposed.*

Various Departments.

Ex. 5. At what rate per cent. will the interest on £1250 for 5 years amount to £265 12*s.* 6*d.*?

		£	*s.*	*d.*
Interest of £1250 for 5 years	=	265	12	6
Interest of £1250 for 1 year =one-fifth of £265 12*s.* 6*d.*	=	£53	2	6

Therefore interest of £1 for 1 year $= \dfrac{\pounds 53\ \ 2\ \ 6}{1250}$

Whence interest of £100 for 1 year $= \dfrac{\pounds 53\ \ 2\ \ 6 \times \cancel{100}}{\cancel{1250}}$

$= \dfrac{\pounds 53\ \ 2\ \ 6 \times 2}{25} = \dfrac{\pounds 106\ \ 5}{25} = 4\frac{1}{4}$ *Ans.*

Various Departments.

12. At what rate per cent. per annum will the interest on £1270 18*s.* 9*d.* amount to £76 5*s.* 1½*d.* simple interest in 2 years?

13. At what rate per cent. simple interest will £320 amount to £360 16*s.* in 3 years?

14. At what rate per cent. simple interest will £951 7*s.* 6*d.* amount to £1141· 13*s.* in 5 years?

War Department.

15. At what rate per cent. per annum simple interest will £300 amount to £414 in 8 years?

138. *Science and Art, Prisons, Post Office, and other Offices.*

Ex. 6. In what time will £2000 amount to £2857 10*s.* at 5¼ per cent. simple interest?

	£	*s.*
The amount =	2857	10
The principal =	2000	
Therefore the interest is	£857	10*s.*

Now, interest of £2000 for 1 year at 5¼ = £105; then £857 10 ÷ 1^{05} = 8 yrs. 2 months. *Ans.*

Various Departments.

16. In what time will £171 7*s*. 6*d*. amount to £217 1*s*. 6*d*. at 3½ per cent?

17. In what time will £1337 12*s*. 1*d*. amount to £1698 15*s*. 1¾*d*. at 6 per cent. simple interest?

139. *Various Departments.*

Ex. 7. What sum of money put out to interest for 3½ years at 4 per cent. will amount to £296 8*s*.?

The interest of £1 for 3½ years at 4 per cent. $=\frac{3\frac{1}{2}\times 4}{100}=\frac{7\times 4}{100\times 2}=\frac{7}{50}$; therefore the amount of £1 at the same rate and for same time $=1\frac{7}{50}$; whence £296 8 ÷ $1\frac{7}{50}=\frac{£\cancel{296}\ \cancel{8}\times 50}{\cancel{57}}=£\cancel{98}\ \cancel{16}\times\frac{50}{\cancel{19}}=£5\ \ 4\times 50$ = £260. *Ans.*

18. What sum will amount to £45 0*s*. 9¾*d*. in a year at 6½ per cent.?

19. What sum of money put out at simple interest for 3 years, at 4½ per cent., will amount to £1602 0*s*. 1½*d*.?

Education and Constabulary Offices.

20. Find the simple interest on £2175 10*s*. for 3 years at 4¼ per cent.

India Office.

21. Find the simple interest on £1923 15*s*. for 2 years and 8 months at 3⅝ per cent.

Proposed in Departments where 'Arithmetic including Vulgar and Decimal Fractions' is to be known.

22. What principal lent at 3½ per cent. simple interest, will amount in 5 years to £1884 19*s*. 10½*d*.?

23. Find the interest at 4¾ per cent. on £1835 8*s*. 4*d*., lent on 11th of March and repaid on 4th August following?

24. At what rate will the interest on £4127 10*s*. amount to £92 17*s*. 4½*d*. in a year?

25. What sum of money put out to interest for 6 years at 3½ per cent. will amount to £259 7*s*. 5*d*.?

26. At what rate per cent. will £1303 6*s*. 8*d*. amount to £1687 14*s*. 10*d*. in 10 years at simple interest?

27. Find the simple interest on £588 6*s.* 8*d.* at 3¼ per cent. for 5 years.

28. In what time will £527 10*s.* amount to £602 13*s.* 4½*d.* at 4¾ per cent. simple interest ?

29. Find the simple interest on £1248 12*s.* at 3⅛ per cent. for 5 years.

30. At what rate per cent. (simple interest) will £2063 15*s.* amount to £2249 9*s.* 9*d.* in 2½ years ?

When partial payments of the principal are made.

Ex. 8. A borrows £300 from B, at 6 per cent., for 2 years, to be paid in instalments or parts. At the end of 6 months he pays £60; 3 months after, £40; 4 months after, £50; 6 months after, £60. What amount in principal and interest will be due to B at the end of the 2 years?

This is several examples of interest in one. In practice we can contract the operation by multiplying by the number of months, as below, and adding the products; then multiplying by the rate (6), and dividing by 1200.

£		£	s.	£
300	for 6 months, at 6=	9	0	300×6=1800
60	first payment			60
240	for 3 months, at 6=	3	12	240×3= 720
40	second payment			40
200	for 4 months, at 6=	4	0	200×4= 800
50	third payment			50
150	for 6 months, at 6=	4	10	150×6= 900
60	fourth payment			60
90	unpaid for 5 mths.=	2	5	90×5= 450
	Total interest	£23	7	4670
	Principal due	£90	0	6
		£113	7 *Ans.*	12,00)280,20
				Interest £23 7

Or,

			£	s.
First payment,	£60	for 6 months=	1	16
Second ,,	£40	,, 9 months=	1	16
Third ,,	£50	,, 13 months=	3	5
Fourth ,,	£60	,, 19 months=	5	14
Principal unpaid	£90	,, 2 years =	10	16
		Interest	£23	7
		Principal unpaid	90	0
			£113	7 *Ans.*

DISCOUNT.

140. DISCOUNT is an allowance made for the payment of money before it becomes due.

If I owe my friend £105, which I am to pay him at the end of a year; he cannot expect me to pay before the proper time, except he pays or abates me a certain sum in consideration of my paying him before he is legally entitled to payment. Supposing that he requires his money presently, he will very probably be satisfied with £100 instead of waiting until one year elapse for £105.

The £100 he gets from me is called the *Present worth* of the £105; the £5 which my friend abates, is called the *Discount*; and, like Interest, it is calculated at a per centage.

Properly speaking the *Present worth* is that sum which, if put to interest at the given rate and for the given time, would amount to the sum due. £100 put to interest for a year, at 5 per cent., will amount to £105; therefore £100 is the *true* present worth of £105 due 1 year hence at 5 per cent.

141. *To find the present worth accurately.*

RULE.—I. Say £100 + its interest at the given rate and for the given time : the given sum :: £100 : the present worth.

Ex. 1. What is the present value of a bill of £550 payable in 2 yrs., at 5 per cent.?

The amount of £100 in 2 yrs., at 5 per cent., is £110, consequently £100 is the *present worth* of £110 due in two years; hence the reason of the statement.

$$£110 : £550 :: £100$$

$$\frac{\cancel{550}\times 10\cancel{0}}{\cancel{110}}=£500\ \textit{Ans.}$$

Or, since the present worth of £110 is £100, the present worth of £1 is $=\frac{100}{1\cdot10}=\frac{10}{11}$; hence £550 $\times\frac{10}{11}=$ £500 *Ans.*

A bill is not *legally* payable in Great Britain and Ireland until 3 days, called 'Days of Grace,' after the time menioned in it. These days must be included in the calculation. When a bill becomes *nominally* due on the 29th, 30th, or 31st, of a month which has not so many days, the last day of the month is taken; therefore the bill will be legally due on the 3rd of the following month. Bills falling legally due on Sunday are paid in Great Britain and Ireland on Saturday.

There is another kind of discount or abatement sometimes made to the purchaser of goods, in consideration of ready money payment. This gives the seller the advantage of a cash payment, while it brings a small reduction in the cost to the buyer.

142. Bankers and merchants invariably calculate the present worth of bills by the following rule:—

Rule.—II.—1°. Find the time from the day on which the bill is discounted, until the day it will be *legally* payable.

2°. The interest of the amount of the bill for this time, and at the given rate, is the discount charged.

Ex. 2. A bill of £120 is drawn on 14th May, at 4 months, and discounted on 3rd August; what ready money must be given for it, at 5 per cent.?

The time from 3rd August till 17th (14th + 3 Days of Grace) September, is 45 days; now the interest of £120 for 45 days is 14*s.* $9\frac{39}{73}$*d.*, the *discount*. Hence £120 − 14*s.* $9\frac{39}{73}$*d.* = £119 5*s.* $2\frac{34}{73}$*d.*, the *present worth*.

Since by this rule the discount of £105 for a year, at 5 per cent., is £5 5*s.*, and £5 by Rule I.; it follows that the discount charged by merchants and bankers is too great. On comparing both results, it will be evident that the interest on any bill, as found by Rule II., exceeds the *true* discount by the interest upon the *true* discount.

Ex. 1. What is the discount on, and the present worth of, a bill of £250 drawn on 13th March at 6 months, discounted on 5th July at 5 per cent.?

This bill will be legally due on 16th September; and from 5th July to 16th September is

July 26
Aug. 31
Sept. 16
73 days

Then the interest of £100 for 73 days at $5\frac{1}{2}$ per cent. $= \frac{1\cancel{00} \times \cancel{73} \times 5\frac{1}{2}}{\cancel{36500}} = \frac{11}{5 \times 2} = \frac{11}{10} = £1\frac{1}{10}$; therefore 100 : $1\frac{1}{10}$ :: 250 : £2 15*s.* the discount; wherefore £250 − £2 15*s.* = £247 5*s.* the present worth.

XXXV.

1. Find the discount on, and the present worth of, a bill of £743 10*s.* drawn on 25th Sept. 1862, at 10 months, discounted 15th March following at 5 per cent.

2. What is the difference between the simple interest and discount on a bill of £1000 drawn at 9 months at $3\frac{1}{2}$ per cent.?

3. How should a hatter sell hats for ready money which he sells at 18*s.* giving a year's credit, reckoning discount at 8 per cent.?

Various Departments.

4. Explain the difference between interest and discount, and find the true present worth of £553 15*s.* due 2 years hence, at $5\frac{3}{8}$ per cent.

5. Explain the difference between interest and discount, and find the present worth of £2674 6*s.* due 3 years hence at $4\frac{3}{5}$ per cent.

6. What is the true discount on £1186 5*s.* 11*d.* due 8 months hence, the rate of interest being $5\frac{3}{4}$ per cent. per annum?

7. Find the discount on £594 14*s.* 9*d.* due 8 months hence, at $5\frac{3}{4}$ per cent. simple interest.

8. Find the discount on £1511 7*s.* 1*d.* payable $3\frac{1}{4}$ years hence, at 4 per cent. simple interest.

9. Find the discount on £2267 0*s.* $7\frac{1}{2}$*d.* payable $3\frac{1}{4}$ years hence, at 4 per cent. simple interest.

10. Required the present value of £430 due 4 months hence at 4 per cent. per annum.

Inland Revenue.

11. The discount on a sum due one year hence at 5 per cent. per annum interest is £15: what is the sum?

COMMISSION &c.

143. COMMISSION, INSURANCE, BROKERAGE, STOCKS, &c., like *Interest* and *Discount*, are calculated at a per centage.

COMMISSION is a per centage paid to an agent or factor for buying or selling goods or transacting other business.

INSURANCE or ASSURANCE is a per centage paid to an individual or public company that undertakes to *pay* the amount insured in case of the entire loss of the property, or to *repair* any accidental injuries it may receive. The *Policy of Insurance* is the legal deed, or document of agreement between the parties concerned. The whole annual payment is called the PREMIUM.

Life Insurance enables a person to secure a certain sum at his death for his family, by paying a company an annual per centage upon that sum, from the time he insures, during the remainder of his life.

Insurance from *sea risk*, or *Marine Insurance*, is a per centage paid on the value of the cargo, for one voyage only, and is usually paid before the vessel sets sail.

BROKERAGE is a small allowance made to a broker for procuring sales, purchasing stock, and transacting other money concerns.

144. *To find the commission &c.*

RULE.—Just proceed as in Simple Interest.

Ex. 1. What is the commission on £426 16*s*., at 5 per cent ?

Here, as $\frac{1}{20}$ of the principal gives the *interest* (now called *commission*) at 5 per cent., we divide by 20 for the answer.

£426 16*s*.
$5 = \frac{1}{20}$ = £21 6*s*. 9$\frac{3}{5}$*d*. *Ans.*

Ex. 2. What is the insurance on £853 12*s*., at £4 6*s*. 8*d*. per cent. ?

In this example we multiply by the rate, and then divide by 100.

£853 12*s*.
4
s. *d*. 3414 8
6 8 = $\frac{1}{3}$ = 284 10
1,00)36,98 18 8
£36 19 9$\frac{1}{2}$ *Ans.*

Ex. 3. What is the brokerage on £7360 16*s.* 8*d.*, at 5*s.* per cent.?

Here $\frac{5s.}{£100}=\frac{5}{2000}=\frac{1}{400}$; hence we divide by 400 for the answer. £7360 16*s.* 8*d.* ÷ 400 = £18 8*s.* 0½*d.* *Ans.* Or, since 5*s.* = £¼, we have,—

$$\begin{array}{rrrr} & £ & s. & d. \\ & 7360 & 16 & 8 \\ \hline s. & & & \\ 5=\frac{1}{4}= & 1840 & 4 & 2 \\ \hline \frac{1}{1,00}= & £18 & 8 & 0\frac{1}{2}\ \textit{Ans.} \end{array}$$

XXXVI.

1. What premium must be paid for insuring goods to the amount of £6500, at 6 per cent.?

2. What is the brokerage on £3964 14*s.*, at 15*s.* per cent.?

3. What is the commission on goods worth £767 14*s.* 7*d.*, at 7½ per cent.?

4. At 2*s.* 6*d.* per cent., what will be the brokerage on £4500?

5. Calculate the brokerage on £3907 10*s.*, at 2*s.* 6*d.* per cent.

6. What is the premium of insurance, at 5 per cent., on goods worth £410 10*s.*?

7. A person aged 26 insures his life for £2340, at £2 9*s.* 6*d.* per cent.; what annual premium must he pay?

8. A merchant insures a cargo whose value is £4500, at 5 per cent.; for what sum should he insure so as to cover both the loss of the cargo and premium? *

9. How much must be insured on goods worth £1805, at 5 per cent., so that the value of the goods and the premium may be repaid in case of loss?

10. What is the premium on a policy of £2475, on a life of 24 years, at £2 2*s.* 4*d.* per cent. for that age?

11. What is the commission on £998 16*s.* 8*d.*, at £7 12*s.* 6*d.* per cent.?

EQUATION OF PAYMENTS.

145. When a person owes several debts payable at different times, to find the time at which the whole might be paid without loss to either party.

RULE.—1°. Multiply each debt by the time which should elapse before it will become due.

---

* If £100 were insured, only £95 would be actually received, since £5 premium was paid; therefore the insurance for £100 will cover a loss of £95 only. Now, if an insurance for £100 covers a loss of £95, what insurance will cover a loss of £4500?

2°. Then divide the sum of the products by the sum of the debts.

Various Departments.

Ex. 1. Find by any rule with which you are acquainted the sum which, paid one year hence, will be an equivalent for £200 due now, and £600 due at the end of two years, the rate of interest being £5 per cent.

The present value of £600 due two years hence is £545$\frac{5}{11}$: then the interest of £200 + £545$\frac{5}{11}$ for a year = £37$\frac{3}{11}$: hence £745$\frac{5}{11}$ (200 + 545$\frac{5}{11}$) + £37$\frac{3}{11}$ = £782 14*s.* 6$\frac{6}{11}$*d.* *Ans.*

Ex. 2. One person owes another £50, payable in 6 months; £60 payable in 8m.; and £90, payable in 4m.: at what time may all be paid without loss or gain?

Here the sum of the products is 1140, which, divided by 200, the sum of the debts, gives 5·7 months.

$$\begin{array}{r} 50 \times 6 = 300 \\ 60 \times 8 = 480 \\ 90 \times 4 = 360 \\ \hline 2{,}00)11{,}40 \\ \hline \end{array}$$

Ans. 5·7 months

XXXVII.

Various Departments.

1. Explain any rule with which you are acquainted for determining 'Equated Time of Payment,' reckoning either simple or compound interest and apply it to find what sum paid two years hence will be an equivalent for £100 due one year hence, and £500 due three years hence, the rate of interest being 5 per cent.

Not Proposed.

2. If I owe my friend £80, payable in 4m.; £70, payable in 6m.; £90, payable in 8m.; and £120, payable in 10m.: in what time may I pay him the whole debt at once?

3. Required the equated time for the payment of three debts: one, £700, payable in 6m.; another of £800, payable in 9m.; and a third of £600, payable in 10m.?

4. A debt is to be paid: ½ at the end of 2m.; ⅓ at 4m.; and the remainder at 12m.: what is the equated time for the payment of the whole?

5. One-half of a debt of £2000 is payable in 10m.; ¼, in 12m.; ⅕, in 16m.; and the remainder, in 20m.; when might the whole be paid at one payment?

THE STOCKS.

146. When the Government of a country cannot obtain a sum of money equal to the exigencies of the State by the ordinary process of taxation, it contracts a Loan at some *fixed* rate of interest, giving the lenders acknowledgments or bonds, implying that the nation is indebted to them for the amount advanced to the Government. These documents or bonds can be sold or transferred from one party to another, like any other kind of property. The interest is payable *half-yearly*, but the time of paying the principal is at the option of the Government. That portion of the revenue from which the interest of the Stocks is paid, is called the FUNDS; and the whole amount of money borrowed by Government is called the NATIONAL DEBT.*

Exchequer Bills are a kind of Promissory Notes issued by the Chancellor of the Exchequer, by the authority of Parliament, to supply some temporary exigency in the funds of the State, upon which interest per cent. per day is allowed until the lender is repaid the principal.

If the rate of interest for money remained *invariable*, the price of £100 stock would always be £100 cash; but as the value of money, like the value of other property, fluctuates according to the abundance or scarcity of money in circulation from time to time, the price of stock *varies* accordingly.

Suppose I have a Government bond for £100 stock, on which I receive 3 per cent. interest. Now, if the general rate of interest rise to 4 per cent., the value of my bond will *fall*, that is, it will be worth less than £100 cash; because a less sum than £100 (£75 in this case) will bring me 3 per cent. interest elsewhere. On the contrary, if the general rate of interest fall to 2½ per cent., the value of my bond will *increase*, as it will take more than £100 (£120 in this case) to bring me £3 interest elsewhere, which I am still paid by Govern ment for my bond. When the interest is fixed, the principal must vary; when the principal is fixed, the interest must vary.*

---

* First borrowed by Charles II., 1660. At the time of the abdication of James II., the national debt was £660,000. William III. established the system.

When trade is dull, and little means of employing capital, there will be much money lying on hand, consequently the rate of interest will be low, and the Funds will rise. But when trade is flourishing, capital will be in demand, the rate of interest will rise, and the Funds will fall. Political changes, &c., also affect the price of stock.

The sales and purchases in the Funds are effected by *an agent* or *stock-broker*, who receives 2*s.* 6*d.* per £100 stock, and, in case of shares, 2*s.* 6*d.* per share. His charge or fee must be *deducted* from the nominal selling price, or *added* to the buying price, according as he is employed in selling or in purchasing.

Sometimes the stocks or shares of railway, and other *companies*, are only gradually paid up, as the necessities of the business require.

147. RAILWAY SHARES.—When a railway is projected a certain number of shares of an amount previously arranged is subscribed. Sometimes the whole sum is at once paid on each share, but more frequently it is paid by instalments or 'calls,' as the necessities of the undertaking may require. For instance, if £50 shares be subscribed, £10 per share may be paid at first, and after a time £5 per share and so on, as required by the progress of the works. Sometimes the whole amount of each share is never paid, as it is always impossible to determine the exact amount required to finish the works. When the original shares have all been paid up, and more money is still necessary, the Company very often give the lenders a guarantee of a certain rate of interest for their shares out of the first of their profits. Shares subscribed in this manner are called *preference*

---

* During the reigns of William III. and Anne, the interest paid for loans by Government varied according to agreement: since that period it has been fixed.

shares, and are usually at 5 per cent. If the profits of the Company amount to more than 5 per cent. the ordinary shares are better than the *preference*, for the latter are only paid according to the guarantee.

The *dividend* is the amount of interest paid on stock or shares: hence when the time of its payment approaches the stocks will rise: hence also the meaning of the letters '*Ex. Div.*' or 'x. d.' (exclusive of dividend), which are attached to the price of shares in newspapers. When £100 stock sells for £100 cash, it is said to be at *par*: when it sells for more than £100 cash, it is at a *premium*; and when it sells for less than £100 cash, it is at a *discount*.

148. PURCHASES AND SALES OF STOCK.

Ex. 1. How much stock can a person purchase in the 3 per cents. at $92\frac{1}{8}$ for £6457 10*s*., brokerage included?

Here £$92\frac{1}{8}+\frac{1}{8}$ (the brokerage) = £$92\frac{1}{4}$, the sum which will purchase £100 stock: then $\dfrac{£6457\ 10 \times 100}{92\frac{1}{4}}$ = the amount which can be purchased for £$6457\frac{1}{2}$ (10*s*. = $\frac{1}{2}$) $=\dfrac{6457\frac{1}{2}\times 100}{92\frac{1}{4}}=\dfrac{\cancel{12915}\times\cancel{4}\times 100}{\cancel{369}\times\cancel{2}}$. (This is found by reducing the two mixed numbers, $6457\frac{1}{2}$ and $92\frac{1}{4}$ to improper fractions, and writing the denominator of the former below and that of the latter above the line) $=\dfrac{\cancel{1435}\times 2\times 100}{\cancel{41}}=35\times 2\times 100=$ £7000. ***Ans.***

By statement, $92\frac{1}{4}$: £6457 10 :: £100 : £7000.

Ex. 2. How much will a person receive for £7000 in the 3 per cents. at $92\frac{5}{8}$, brokerage $\frac{1}{8}$ per cent.?

Every £100 stock will realise £$92\frac{5}{8}-\frac{1}{8}$ (the brokerage) = £$92\frac{1}{2}$: then $\dfrac{£7000\times 92\frac{1}{2}}{100}=\dfrac{\cancel{7000}\times 185}{\cancel{100}\times\cancel{2}}=35\times 185=$ £6475. ***Ans.***

XXXVIII.

Find the amount of stock which can be purchased by investing:—

1. £6783 in the 3 per cents., at 76.
2. £6129 in the 4 per cents., at $85\frac{1}{8}$.
3. £5048 in the $3\frac{1}{2}$ per cents., at $78\frac{7}{8}$.
4. How many shares can be purchased in a railway for £503 10*s*., at $26\frac{1}{2}$?
5. How many Bank shares can be purchased for £4600 at $28\frac{3}{4}$, brokerage included?

Find how much sterling money will be realised by selling :—

6. £1870 10*s*. 5*d*. in the 3¾ per cents. at 87.
7. £1200 in the 4 per cents. at 92⅝.
8. £1575 in the 3 per cents. at 75⅜.
9. £4768 15*s*. bank stock (£10 shares) at 35½, brokerage included.
10. £6806 19*s*. 3*d*. in railway shares, at 76.

149. INCOME OF INVESTMENTS.

Ex. 3. What yearly income will a person secure by investin £1991 15*s*. in the 3 per cents., at 96⅜?

Here 96⅜ gives £3 income, whence 96⅜ : 1991¾ : £3

$$\frac{771}{8} : \frac{7967}{4} :: 3.$$

$$\frac{\cancel{8}}{771} \times \frac{7967}{\cancel{4}} \times 3 = \frac{47802}{771} = \pounds 62. \quad \textit{Ans.}$$

Ex. 4. How much should a person invest in the 3½ per cents. at 78 to secure an income of £154?

Every £78 invested secures an income of £3 10*s*.: hence,

$$\pounds 3\ 10 : \pounds 154 :: 78$$

$$\frac{2}{\underset{1}{\cancel{7}}} \qquad \frac{2}{\underset{44}{\cancel{308}}} \times 78 = \pounds 3432. \quad \textit{Ans.}$$

What income will arise from investing :—

11. £2578 9*s*. 4½*d*. in the 3 per cents., at 72⅛.
12. £2356 10*s*. 5*d*. in the 3 per cents., at 73⅝.
13. £3963 4*s*. 9½*d*. in the 4 per cents., at 98⅞.
14. £4473 16*s*. 8*d*. in railway shares, paying 5 per cent., at 110¾
15. £1128 8*s*. 9*d*. in Royal Banks, at 28⅝, including brokerage, paying 2½ per cent.

150. RATES OF INTEREST.

Ex. 5. What is the rate of interest when the 3 per cents at 80 ?

Here £80 gives £3: hence £100 will give

$$\frac{100 \times 3}{80} = \frac{300}{80} = 3\tfrac{3}{4} \ \textit{Ans.}$$

What rate of interest per cent. per annum will be received by ing :—

16. In the 3 per cents. at 84 : 4 per cents. at 90.
17. In the 3½ per cents. at 88 : 4 per cents. at 96.
18. In railway 5 per cent. preference shares, at 105⅝.
19. In £10 shares Bank stock, paying 12 per cent., at 30.

151. TRANSFERS.

Ex. 6. A person transfers his capital from the $3\frac{1}{2}$ per cents. at 77 to the 4 per cents. at 89: find the increase or decrease per cent. in his income.

Every £77 gives him £3 10*s*. income, before the transfer: whence $3\frac{1}{2} : 4 :: 77 : 88$: therefore £88 would give £4 in the $3\frac{1}{2}$ per cents., but £89 is paid in the 4 per cents. Hence we have $89 - 88 =$ £1 loss on £4 income.

∴ the loss on £100 income will be £25, that is, his income will be decreased 25 per cent.

Ex. 7. In a certain railway, the total amount in shares subscribed is £80000, of which there is £10000 preference shares at 5 per cent: the profits for the year are £3475: what per centage will be paid upon the ordinary shares?

£10000 preference at 5 per cent. = £500: then £3475 − £500 = £2975 the sum to be paid on the 70000 ordinary shares.

Therefore $70000 : 100 :: £2975 : 4\frac{1}{4}$. *Ans.*

Various Departments.

Ex. 8. A person sells out of the 3 per cents. at 96, and invests his money in railway 5 per cent. stock at £106 13*s*. 4*d*.: find how much per cent. his income is increased.*

Here $\frac{96 \times 100}{3} = £3200$, the sum which, if invested in the 3 per cents., will give an income of £100 per annum. The same sum invested in 5 per cent. stock at £106 13*s*. 4*d*. ($106\frac{2}{3}$), will return an income of $\frac{3200 \times 5}{106\frac{2}{3}} = \frac{\cancel{320}0 \times 5 \times 3}{\cancel{320}} = 10 \times 5 \times 3 = £150$: hence £150 − £100 = 50, the increase; that is, 50 per cent.

Ex. 9. What must be the market value of 5 per cent. stock, in order that, after deducting the income-tax of 5*d*. in the pound, it may yield $4\frac{1}{2}$ per cent. interest?

Here £1 gross income gives (the tax being paid) 19*s*. 7*d*. net; the gross income = the net income + $\frac{5d.}{19s.\ 7d.}\left(=\frac{5}{235}=\frac{1}{47}\right)$, hence £4 10*s*. net will require $90s. + \frac{90}{47}s.$ gross $= 91\frac{43}{47}s.$ Now, if 100 produces $91\frac{43}{47}s.$ what sum will produce £5?

---

* Since £96 stock = £3 income, £1 income = 96 ÷ 3 = £32 stock. Now £32 invested in the railway shares will bring an income of $\frac{£32 \times 5}{£106\frac{2}{3}}$ = £1 10*s*.: whence the gain is 10*s*. on £1 income = 50 per cent. as before.

We may state,

$$91\frac{43}{47}s. : 100s. :: £100 : \frac{470000}{4320} = \frac{5875}{54} = £108\tfrac{43}{54} = £108\ 15s.\ 11\tfrac{1}{9}d.\ \textit{Ans.}$$

Education and Constabulary Offices.

20. When the 3 per cents. are at 89¾, how much can be bought for £540, allowing for commission ⅛ per cent. upon the stock bought?

21. A person who has £2954 in the 3 per cents. at 80½, transfers his capital into the 4 per cents. at 98: find the alteration in his income.

22. When the 3 per cents. are at 91¾, find how much can be bought for £540, allowing ⅛ per cent. for commission upon the stock bought.

23. A person sells £5000 consols at 94⅞, and on their rising he sells £5000 more at 95⅝; on their again rising, he buys back the whole £10000 at 96. What does he lose on the transaction?

24. A person sells out of the 3 per cents. at 96, and invests his money in railway 5 per cent. stock at par: find how much per cent. his income is increased?

Inland Revenue.

25. A person sells £7500 railway stock, which pays 6 per cent. at 120, and invests half the proceeds in the 3 per cent. consols at 96, and the other half in the South Eastern stock, which pays 5½ per cent. at 90. How is his annual income affected?

Various Departments.

26. A person who has £1475 in the 3 per cents. at 75¼, transfers it to the 5 per cents. at 110⅝: what is the alteration in his income?

27. A person invests £9075 in the 3 per cents. at £90¾, and on their rising to 91 transfers it to the 3¼ per cents. at 93½: how is his annual income affected?

28. If the 3½ per cents. be at 91, how much must I invest in them in order to have a yearly income of £932, after paying 7*d.* in the pound income-tax?

29. A person invests £9075 in the 3 per cents. at 90¾, and on their rising to 91 transfers it to the 3½ per cents. at 97⅛: what increase does he make thereby in his annual income?

30. A person sells £6000 3 per cent. consols at 92¼, and invests this sum in railway stock, paying 5½ per cent. at 103⅛. Find how his income is affected (neglecting fractions of a penny).

Admiralty.

31. A person sells £8000 3 per cent. consols at 92⅞, and invests the proceeds in railway shares (paying 5 per cent.) at 102: how much is his income increased?

Various Departments.

32. Compare the 3 per cents. at 89⅜, and railway shares paying 5 per cent. at 102⅝.

PROFIT AND LOSS.

152. When a person sells an article for the same money which he paid for it, he has no profit: when he sells it for twice its cost he gains 'cent. per cent.' If an article be sold for half as much more as was paid for it, the gain is 50 per cent., that is, £50 on every £100 of outlay: if sold for one-fourth as much more than its cost the gain is 25 per cent., and so on. It is also evident that, if half the number of articles be sold for what the whole cost, the gain on them is 'cent., per cent.' If three-fourths the number of articles be sold for what the whole cost, the gain is 33⅓ per cent.; if ⅘ of the number, the gain is 25 per cent.: if ⅗, it is 66⅔ per cent., and so on.

The method of solving questions in this rule will be understood from the following examples.

Various Departments.

Ex. 1. A person having £5704 18*s.* 4½*d.* lays out one-third in goods, which he sells for £2316 5*s.* 10*d.*: how much has he at last, and how much has he gained?

	£	*s.*	*d.*	
From	5704	18	4½	his capital
Take ⅓ of it =	1901	12	9½	
Cash remaining =	3803	5	7	
Selling-price of goods =	2316	5	10	
First cost of same =	1901	12	9½	
His gain =	414	13	0½	
Original capital =	5704	18	4½	
	£6119	11	5	sum he has at last. *Ans.*

Education Office (London).

Ex. 2. If 20 lbs. 4 oz. of tea at 5*s.* 10*d.* per pound be mixed wi 46 lbs. 6 oz. at 4*s.* 3*d.* and 35 lbs. 6 oz. at 6*s.* 8*d.* and the mixture sold at 6*s.* per pound, what will be the gain or loss, and what the ga or loss per cent. ?

	£	*s.*	*d.*
20 lbs. 4 oz. @ 5*s.* 10*d.* =	5	18	$1\frac{1}{2}$
46 lbs. 6 oz. @ 4*s.* 3*d.* =	9	17	$1\frac{1}{8}$
35 lbs. 6 oz. @ 6*s.* 8*d.* =	11	15	10
102 lbs. prime cost of =	27	11	$0\frac{5}{8}$
Again, 102 lbs. @ 6*s.* =	30	12	
Subtract first cost and the gain is	£3	0	$11\frac{3}{8}$

For the gain per cent. we have,

$$£27\ 11\ 0\tfrac{5}{8} : 100 :: £3\ 0\ 11\tfrac{3}{8}$$

		8			8
220	8	5	24	7	7
20			20		
4408			487		
12			12		
52901			5851		
			100		

52901)585100 (£11 1*s.* $2\frac{24746}{52901}$*d.* *Ans.*
581911

3189
20

63780(1
52901

10879
12

130548(2
105802

24746

52901

Customs and Excise.

Ex. 3. By selling cloth at the rate of 12*s.* per yard £1! was gained. What per cent. would be gained when it was s rate of 10*s.* 6*d.* per yard ?

Here £115 : 100 :: 12*s.* : $\frac{\cancel{100} \times 12}{\cancel{115}} = \frac{20 \times 12}{23} = \frac{240}{23} = 10\frac{10}{23}$*s.* the prime cost per yard.

Now $10\frac{1}{2} - 10\frac{10}{23} = 10\frac{23}{46} - 10\frac{20}{46} = \frac{3}{46}$*s.* the gain per yard.

Whence $10\frac{10}{23} : 100 :: \frac{3}{46}$

Multiply the first and third by 46 (31).

$$\begin{array}{ccc} 46 & & 46 \\ \hline 480 & : 100 :: & 3 \end{array}$$

$$\frac{10\cancel{0} \times \cancel{3}}{\cancel{480}} = \frac{10}{16} = \frac{5}{8} = 12s.\ 6d.\ \textit{Ans.}$$

XXXIX.

1. How must butter be sold which cost £3 14*s.* per cwt. to gain 16 per cent.?

2. A merchant sold 200 yards of cloth for £75, and gained 15 per cent.; what did the cloth cost him per yard?

3. If 10 per cent. be gained by selling butter at £3 6*s.* 8*d.* per cwt., what will be gained per cent. by selling it at 7½*d.* per lb.?

4. Bought 4 packs wool, at £130 10*s.* each, and sold the whole at 20 per cent. profit; what was the selling price per pack?

5. A person sells a horse for £60, and loses 20 per cent., while he should have gained 12 per cent.; how much did he pay for the horse, and what is his entire loss?

6. If I sell cloth at 15*s.* per yard, and lose 10 per cent., what should be the selling price per yard to gain 18 per cent.?

7. What is gained by selling 178 cwts. 2 qrs. 14 lbs. of beef at 6½*d.* per lb., which was bought at £2 13*s.* 4*d.* per cwt.?

8. By selling 178 cwts. 2 qrs. 14 lbs. at 6½*d.* per lb., I gain 13¾ per cent.; what was the first cost of the whole?

9. How much merchandize, at 7½*d.* per lb., at a gain of £6 13*s.* 4*d.* per cent., must be sold to gain £63 12*s.* 3*d.*?

10. Bought 500 stones of sugar upon which I gained £14 11*s.* 8*d.*, and found I had gained 12½ per cent.; what was my buying and selling price per stone?

War Department.

11. If 23 lbs. at 2*s.* are mixed with 27 lbs. at 3*s.* and the mixture is sold at 2*s.* 9*d.* per lb., what is the total gain, and what is the gain per cent. on the money laid out?

Various Departments.

12. By selling an article for £9 10*s.* the seller loses 5 per cent. on his outlay; what would be his loss or gain per cent, if he sold it for £11 17*s.* 6*d.* ?

13. A grocer buys 3 cwts. of sugar at 5*d.* per pound, and 7 cwts. at $6\frac{1}{3}d.$; he sells $5\frac{1}{2}$ cwts. at $5\frac{1}{2}d.$ per pound; at what rate per pound must he sell the remainder in order to make 50 per cent. on his whole outlay ?

14. A person buys teas at 3*s.* and 4*s.* the pound, and mixes them in the proportion of 4 to 7; what will he gain per cent. by selling the mixture at 4*s.* 2*d.* per pound ?

AVERAGES AND PERCENTAGES.

153. 'An Average is a mean or medium of two or more quantities,' and is usually found by dividing the sum of the quantities by the number of them. A *percentage* is a comparison of the gains or losses in trade, the increase or decrease in population, &c., with 100 as a common standard.

Education and Constabulary Offices.

Ex. 1. Find the average of $17\frac{1}{2}$, $25\frac{1}{4}$, $96\frac{3}{8}$, 10, 0, $42\frac{3}{4}$, 56; and express the fractional part decimally.

$$\begin{array}{r} 17\frac{1}{2} \\ 25\frac{1}{4} \\ 96\frac{3}{8} \\ 10 \\ 0 \\ 42\frac{3}{4} \\ 56 \\ \hline 247\frac{7}{8} \end{array} = 247\cdot875$$

Then $247\cdot875 \div 7$ the number of quantities $= 35\cdot410\dot{7}1428\dot{5}$ *Ans.*

Ex. 2. In a certain office there is one person receiving £2000, two who receive £1100 each, six who receive £400, twelve who receive £200 each, and eighteen who receive £90 each; what is the average income of the persons employed in the office ?

The total salaries of all is £10620, and the total number of persons employed is 39; hence the reason of the annexed operation.

$$\begin{array}{rl} 1 \times 2000 = & 2000 \\ 2 \times 1100 = & 2200 \\ 6 \times 400 = & 2400 \\ 12 \times 200 = & 2400 \\ 18 \times 90 = & 1620 \\ \hline 39 \quad 39) & 10620 \end{array}$$

78 (£272 6s. $1\frac{11}{13}$d. *Ans.*

282
273

90
78

12
20

39)240(6
234

6
12

39)72(1
39

$\frac{33}{39}$ $(=\frac{11}{13})$

India Office.

Ex. 3. In 1858 the value of £100 tithe rent charge, reckoned on the average price of corn in the 7 years immediately preceding, was £105; in 1859, reckoned in the like way, it is 3 per cent. more. If it were reckoned on the price in 1851 only, it would be but £69. What would it be if reckoned on the price in 1858 only?

seven years.

1851
1852
1853
1854 } £105 average.
1855
1856
1857

Total for these seven years = 105 × 7 = £735. For the year 1851 it was £69; therefore for the other six years the total was £735 − £69 = £666 (1852—57 inclusive).

Again, 3 per cent. more than £105 = $\frac{105 \times 103}{100}$ = £108 3s. the average for the seven years, which multiplied by 7 gives the total for these 7 years.

$$\left.\begin{array}{l}1852\\1853\\1854\\1855\\1856\\1857\\1858\end{array}\right\}\pounds108\ 3s. \times 7d. = \pounds757\ 1s.$$

whence, £757 1*s.* − £666 = £91 1*s.* *Ans.*

Education Offices (London and Dublin), and Constabulary Office.

Ex. 4. The populations of 3 towns in the year 1841 were 20325, 41304, and 6117; and in the year 1851 they had increased respectively 9, 10, and 12 per cent.; find the average population of the 3 towns in 1851.

The population of the first in 1851 was $\frac{\cancel{20325} \times 109}{\cancel{100}} = \frac{813 \times 109}{4} =$ 22154·25.

The population of second was $\frac{41304 \times 110}{100} = 413{\cdot}04 \times 110 =$ 45434·40.

The population of the third was $\frac{6117 \times \cancel{112}}{\cancel{100}} = \frac{6117 \times 28}{25} =$ 6851·04.

$$\begin{array}{rl}\text{Then } & 22154{\cdot}25\\ & 45434{\cdot}4\\ & 6851{\cdot}04\\ \hline 3) & 74439{\cdot}69 \text{ Total population in 1851.}\\ \hline & 24813{\cdot}23 \text{ Average pop. in 1851. } \textit{Ans.}\end{array}$$

Ex. 5. From the following table relating to Ireland:—

Year.	Number of Roman Catholics.	Number of Episcopalians.
1834	6436060	853160
1861	4505265	691872

Find (1) the decrease per cent. of Roman Catholics; (2) that of Episcopalians; (3) the percentage proportion of members of the Established Church, in 1834 and also in 1861, to the Roman Catholics.

From 6436060
Take 4505265

1930795 = decrease of Roman Catholics; whence decrease per cent. $=\frac{1930795\times 100}{6436060}=29\cdot9996...$

From 853160
Take 691872

161288 = decrease of members of the Established Church; whence decrease per cent. $=\frac{161288\times 100}{853160}=18\cdot90477...$

For the percentage proportion in 1834

$6436060 : 853160 :: 100 : 13\cdot25593...$

and in 1861, $4505265 : 691872 :: 100 : 15\cdot3569...$

XL.

Education Office (Dublin)

1. If by selling wine at 15*s.* per gallon I lose 10 per cent., at what price must it be sold to gain 15 per cent.?

The following have been proposed in Education Offices (England and Ireland) and the Constabulary (Ireland).

2. Find the average of 13, 27, 0, 32, 106, 86; and express the fractional part decimally.

3. The population of five parishes being 1236, 452, 364, 516, and 3430 respectively, find what the population of a sixth parish must be in order that the average population of the six may be 1256·5.

4. A person has $\frac{1}{8}$ of a ship worth £3484 which is insured for $91\frac{2}{3}$ per cent. of its real value; what would he lose in case of the ship being lost?

5. The population of 3 towns in the year 1841 were 21326, 42324, and 6706; and in the year 1851 it was found that the first two had increased 12 and 10 per cent. respectively, and the last decreased 18 per cent.; find the average population of the 3 towns in the year 1851.

6. If a person sells 22 articles for the same money which he paid for 36, what does he gain per cent. on his outlay?

7. Find the average of 13, 27, 0, 46, 72, 86; and express the fractional part decimally.

8. If goods which were bought at £2 5*s.* 10*d.* per cwt. be sold at £2 11*s.* 4*d.* what is the gain per cent.?

9. If by selling an article at £1 1*s.* 9*d.* a pound I gain 16 per cent. on my outlay, what was its prime cost?

10. What is the premium on a policy of insurance for £9626 11*s.* 3*d.* at £2 12*s.* per cent.?

11. A grocer buys 3 cwts. of sugar at 5*d.* per pound and 7 cwts. at 6½*d.*; he sells 5½ cwts. at 5½*d.* per lb.; at what rate must he sell the remainder to make 25 per cent. on his outlay?

12. An army lost 18 per cent. of its strength by disease and desertion, and then lost 14 per cent. of the remainder in battle; the number then remaining was 84624; of how many did the army originally consist?

13. In 1841 the population of Great Britain was 21476000, and that of Ireland 7310000. In 1851 the former had increased 8·45 per cent. and the latter had decreased 12·5 per cent. Find the increase per cent. in the population of the whole kingdom.

14. A ship valued at £14500 is insured at £3 10*s.* per cent., and her cargo, valued at £32000, is insured at £5 per cent. Find the whole cost of insurance.

15. Having given the following table:

Length of Railway in England in 1840.	Number of		
Miles.	First Cls. Passengers.	Second Class.	Third Class.
4355·5	6186899	18844152	24848312

Find (1) the average number per mile of each class.

16. (2) the percentage proportion of each of the three classes.

INVOLUTION.

154. 'If a number is multiplied by itself, the product is called the *second power* or *square* of that number.' Thus $6\times6=36$; 36 is the *second power* or *square* of 6: and since $10\times10=100$; 100 is the square of 10. The numbers 6 and 10 are said to be *involved*, and are called *roots* in reference to their powers: 6 is the *square root* of 36.

If the *square* or *second power* of any number be multiplied by its *root*, the product is the *third power* or *cube* of that number. If the *third* power be multiplied by the root, the product is the *fourth* power, and so on.

Thus, 36×6 $(6 \times 6 \times 6) = 216$, the *third* power of 6; and 4×2 $(2 \times 2 \times 2) = 8$, the *third* power or *cube* of 2; also $8 \times 2 = 16$, the *fourth* power of 2.

The number of times a root is to be taken as factor is indicated by an *index*, which also points out the *power* or *degree* to which the root is to be involved. Thus we say $6^2 = 36$; $2^2 = 4$; and $6^3 = 216$.

155. To raise a number to any power, multiply the number by itself, and this product again by the number, and so on, until the number has been used as multiplier a time *less* than the index. The second power of a number when multiplied by itself gives the fourth power. The second power multiplied by the third gives the fifth, and so on according to the sum of the indices of the powers multiplied.

Ex. 1. Raise 8 to the fifth power.

Here, $8 \times 8 = 64$, second power; $64 \times 8 = 512$, the third power; $512 \times 8 = 4096$, the fourth power; $4096 \times 8 = 32768$, the fifth power.

With fractions, raise their terms to the required power, and write the result as a fraction. Or, reduce vulgar fractions to decimals, and raise the decimal to the required power. It will generally be most convenient to reduce mixed numbers to improper fractions, before multiplying.

Involve the following to the powers indicated by their indices:—

XLI.

1. 22^3, 30^2.
2. ${\cdot}04^4$, $3{\cdot}5^2$.
3. 12^5, $1{\cdot}05^5$.
4. 44^3, $\frac{2}{3}^3$.
5. 36^3, 20^3.
6. 367^2, 19^4.

EVOLUTION.

156. *Evolution* is the finding of *roots* when any power of these roots is given.

The second powers of the first ten numbers are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100; therefore all *square* numbers, whose roots can be *exactly* found, must end either in 1, 4, 5, 6, 9, or 0.

The *sign* $\surd$, when placed before a number denotes that the square root of that number is to be taken: thus, $\sqrt{36}=6$; $\sqrt{100}=10$.

The extraction of any *other root* is indicated by writing its index within this sign: thus, for the extraction of the third root, write, $\sqrt[3]{\ }$; $\sqrt[3]{8}=2$; and $\sqrt[3]{64}=4$.

157. *To find the square root of a number.*

RULE.—1°. Divide the number into periods of two figures each, commencing at the units' place, proceeding towards the *left* in whole numbers, and towards the *right* in decimals.

2°. Find a square number equal to, or next less than the left-hand period; write it under the period, and place its root as a quotient figure.

3°. From the left-hand period, take the number written under it, and annex the next period to the remainder for a dividend.

4°. For a divisor, write a 0 after twice that part of the root already found; find how often this number is contained in the dividend; write the quotient figure in the *place* of the 0 in the divisor, as well as in the quotient.

5°. Then multiply the divisor, thus changed, by the last figure of the quotient, and write the product under the dividend and subtract.

6°. Bring down the next period to the remainder for a new dividend, and double the quotient for a new divisor as before. In finding the *decimals* of a root which has no decimal figures in the power, annex 00 for period: in circulators, bring down two of the circulating figures.

War Department.

Ex. 1. Extract the square root of 106929.

In this example, first divide the number into periods and we get 10 for the *left-hand* period. The next less *square* number to which is 9; the root of 9 is 3, which is placed in the quotient. Next take 9 from 10, and bring down 69, the *next period*, to the remainder. For a divisor double 3, which gives 6, to which annex 0; then try how often 60 is contained in 169, and place the next quotient figure, 2, both in quotient and divisor, multiply 62 by 2, subtract 124 from 169, and bring down the next period. For a new divisor, multiply 32 by 2 = 64, which write, 640 for trial divisor, as before.

```
                       10,69,29
Trial divisor,  60     9
                ——     ——(327 Ans.
     Divisor,  62)    169
Trial divisor, 640    124
               ———    ———
    Divisor,  647)    4529
                      4529
                      ————
```

Ex. 2. Find the square roots of 81830116 and 197·739844.

In both these examples there is a 0 in the root, which is written in *divisor* as well as quotient.

```
          81,83,01,16                 1,97·73,98,44
          81                          1
          ——————(9046 Ans.            ————(14·062 Ans.
  1804)   8301                  24)   97
          7216                        96
          ————                        ————
 18086)   108516              28·06)17398
          108516                      16836
          ——————                      —————
                              28122)  56244
                                      56244
                                      —————
```

Ex. 3. Extract the square root of 5 and that of 3⅔.

```
        5     (2·23606...                 3⅔ =
        4                                 3·6̇
       ——                                 1
  42) 100                                 ——(1·9148...
       84                          29)   266
      ———                                261
 443) 1600                               ———
      1329                        381)   566
      ————                               381
4466) 27100                              —————
      26796                      3824)   18566
      —————                              15296
447206) 3040000                          ——————
        2683236                 38288)   327066
        ———————                          306304
         356764...                       ——————
                                         ...
```

In extracting the square root of $3\frac{2}{3}$ we could have said $3\frac{2}{3}=\frac{11}{3}$; then since $\frac{11}{3}=\frac{11\times3}{3\times3}$ we have $\sqrt{\left(\frac{11\times3}{3\times3}\right)}=\frac{\sqrt{33}}{3}$: hence we could find $\sqrt{3\frac{2}{3}}$ by finding the square root of 33 and dividing it by 3.

XLII.

India Board.

1. Extract the square root of 2854·7649.

Competitive Examinations in Science and Art, &c.

2. Extract the square root of 406278·76.
3. " " $262\frac{11}{25}$.

Various Departments.

4. Find the square root of $5\frac{31}{225}$.
5. " " $33\frac{144}{225}$.
6. " " $514\frac{185}{256}$.
7. " " 49660209.
8. " " 7·0032.
9. " " 1293785.

158. *Extraction of the third or cube root.*

Ex. 1. Find the cube root of 108190397125.

$$\begin{array}{rrl}
 & & 108,190,397,125 \\
 & & \underline{\;64\;\;\;\;\;}(4765 \\
3\times4^2= & 4800 & 44190 \\
3\times4\times7= & 840 & \underline{39823} \\
7^2= & \underline{49} & 4367397 \\
 & 5689 & \underline{4027176} \\
 & \underline{49} & 340221125 \\
 & 662700 & \underline{340221125} \\
3\times47\times6= & 8460 & \\
6^2= & \underline{36} & \\
 & 671196 & \\
 & \underline{36} & \\
 & 67972800 & \\
3\times476\times5= & 71400 & \\
5^2= & \underline{25} & \\
 & 68044225 &
\end{array}$$

We first divide the number into *periods* of *three* figures each, just as in Numeration. Then look for the nearest cube root of 108, the left-hand period, which is 4; after subtracting its cube we have 44 as remainder. To this we bring down the *next* period, 190; then take 3 *times* the square of 4, making 48, to which we annex 00, for the *trial divisor*: then, 44190 divided by 4800 gives 7, the *next* figure of the root. For the complete divisor take 3 times the previous part of the root and multiply it by 7 = 84, to which annex 0; to this add

$7^2=49$: these *three* numbers added give 5689, which multiply by 7, and placing the product under the dividend subtract, and bring down the next period, and so on. For the *next divisor* add 840 and 49 and 5689 and 49, the four numbers connected by the bracket, to which add 00, as before, for the next *trial divisor*, 662700. Every *successive* divisor is found in the same manner, and the work will stand as in the example.

In extracting the cube root of fractions we proceed in a similar manner as in finding the square root; add three cyphers for decimal figures, and three circulating figures if the fraction be a circulator.

XLIII.

Extract the cube root of:—

1. 356853433357.
2. 881234·287298.
3. $673834153\dot{6}63$.
4. $5164\frac{309}{343}$.

Constabulary.

5. Extract $\sqrt[3]{25}$ to 3 places of decimals.

*Various Departments.**

6. Extract the cube root of 134217728.
7. " " 12167.
8. " " 228099131.
9. " " 493039.

India Office.

10. Extract the cube root of 1194389981.

---

* Almost always at Competitive Examinations.

The rules for the extraction of the square and cube roots just given, enable us to find the fourth, sixth, eighth, &c. roots.

To find the 4th root, take the square root of the square root.

To find the 6th root, take the square root of the cube root.

To find the 8th root, extract the square root three times; and for the 9th root, take the cube root three times. From the equations below the student will at once understand the reason.

$6^4=6^2\times6^2$ Or, $x^4=x^2x^2$
$6^5=6^3\times6^2$ $x^5=x^3x^2$
$6^7=6^3\times6^2\times6^2$ $x^7=x^3x^2x^2$
$6^8=6^2\times6^2\times6^2$ $x^8=x^2x^2x^2$
$6^9=6^3\times6^3\times6^3$ $x^9=x^3x^3x^3$ and so on.

PROGRESSIONS.

159. When several numbers follow one another in regu- ır order by the addition or subtraction of the same number, hey form a *series* or *Arithmetical Progression.*

Thus 5, 7, 9, 11, &c., each successive number increasing ɔy 2, form an increasing series: and 12, 9, 6, &c. each ıccessive number being 3 less than the preceding, form a lecreasing series. The number added or subtracted is alled the *common difference.*

160. *To find the sum of an Arithmetical Series.*

Rule.—Multiply the sum of the extremes (the first and st terms) by half the number of terms.

Ex. 1. If the greatest term be 27, the least 3, and the number of rms 7, what is the sum of the series?

27
3

Sum of extremes = 30 ; then $\frac{30 \times 7}{2} = 15 \times 7 = 105$ *Ans.*

161. *To find the Common Difference.*

Rule.—Divide the difference of the extremes by one less ıan the number of terms.

Ex. 2. The extremes are 2 and $39\frac{1}{2}$ and the number of terms 13, hat is the common difference?

$39\frac{1}{2}$
2

Difference of extremes = $37\frac{1}{2}$

$13 - 1 = 12$ one less than number of terms ; then $\frac{37\frac{1}{2}}{12} = 3\frac{1}{8}$ the com- ıon difference, and the series will be 2, $5\frac{1}{8}$, $8\frac{1}{4}$, &c.

XLIV.

1. Find the sum of the series when the least term is 3, the greatest 0, and the number of terms 10.

2. If the extremes be 6 and 42 and the number of terms 7, wl the common difference?

3. The least term is 3, the greatest 9; find the sum of the series, number of terms being 9?

4. In the last question what is the common difference?

5. The sum of the series is 54 and the extremes 7 and 11; what is tl common difference?

6. Find the sum of 15 terms of the series 5·2, 6·1, 7, &c.

7. Find the sum of 22 terms of the series 1, $1{\cdot}\dot{3}$, $1{\cdot}\dot{6}$, $1{\cdot}\dot{9}$, &c.

8. Determine the common difference of a series the sum of which is $16{\cdot}\dot{7}1428\dot{5}$, the number of terms 9, and the last term 3.

162. In a *Geometrical Progression* the numbers increase or decrease by the multiplication or division by a number called the *ratio*. Thus 3, 6 (3×2), 12 (6×2), 24 (12×2) is an increasing series. On the contrary 64, 32 $(64 \div 2)$, 16 $(32 \div 2)$, 8 $(16 \div 2)$ is a decreasing series.

163. *To find any term of a Geometrical Series.*

RULE.—Raise the ratio to a power indicated by one less than the number of terms as index, and multiply (divide in a decreasing series) the least term by the product.

Ex. 2. The common ratio is 3, the number of terms is 7, and one extreme is 9; to find the other.

Here $3^6 = 729$; then $729 \times 9 = 6561$. *Ans.*

In this example the fourth term will be $3^3 \times 9 = 27 \times 9 = 243$: in like manner any other term may be found.

164. *To find the sum of a Geometrical Series.*

RULE.—Divide the difference of the extremes by one less than the ratio; then add that quotient to the greater extreme for the sum of the series.

Ex. 3. Given the extremes 7 and 54432, and the ratio 6; required the sum of the series.

Here $54432 - 7 = 54425$: then $54425 \div 5 = 10885$, and $10885 + 54432 = 65317$. *Ans.*

9. The extremes are 3 and $117\frac{3}{16}$, the common ratio $2\frac{1}{2}$, what is the sum of the series?

10. In the above what is the fourth term?

11. The first term is 4, the number of terms 6, and the common ratio 4; sum the series.

12. What is the fifth term in the last example?

13. The common ratio is ·3, the first term 27, and the number of terms 7, what is the sum of the series?

14. What is the fourth term in the last question?

COMPOUND INTEREST.

165. *To find the Compound Interest.*

Rule.—I.—1°. Add the interest due at the end of each year to the principal that produced it.

2. Consider this amount as principal for the next year, to which add its interest, as before; and so on, until the end of the given time.

Ex. 1. What is the amount of £760 in 4 years, at 5 per cent., compound interest?

By Decimals.	£760 *is the first principal.*
760	$\frac{1}{20}$ = 38 *first year's interest.*
760 × ·05 = 38	
	798 *is the second principal.*
798	$\frac{1}{20}$ = 39 18*s. second year's interest.*
798 × ·05 = 39·9	
	837 18*s. is the third principal.*
837·9	$\frac{1}{20}$ = 41 17 $10\frac{4}{5}$ *third year's interest.*
837·9 × ·05 = 41·895	879 15 $10\frac{4}{5}$ *is the fourth principal.*
	$\frac{1}{20}$ = 43 19 $9\frac{27}{50}$ *fourth year's interest.*
879·795	
879·795 × ·05 = 43·98975	£923 15*s.* $8\frac{17}{50}$*d. amount in four years.*
923·78475 =	
£923 15*s.* 8·34*d. Ans.*	

The interest being 5 per cent. we take $\frac{1}{20}$ of the principal. It is necessary to remark that when the payments are made *half-yearly* we must substitute the word 'half-year' for 'year' in the rule.

166. Rule.—II.—1°. Find the amount of £1 for a year, at the given rate, by simple interest.

2°. Involve this amount to the power indicated by the number of payments as index, and multiply the result by the principal for the compound interest.

The same example, £760 for 4 years, at 5 per cent.

Here, £1 amounts to £1·05 at the end of a year, for 1 × ·05 + 1 = 1·05. Then $1·05^4$ = 1·21550625, and £760 (the principal) × 1·21550625 = 923·78475 = £923 15*s.* 8·34*d.* the *Ans.*, as above. The compound interest is £923 15*s.* 8·34*d.* − £760 = £163 15*s.* 8·34*d.*

Various Departments.

Ex. 2. What will be the amount of £825 at the end of two years, at 4 per cent. per annum compound interest, the interest accruing due half-yearly?

The amount of £1 in half a year, at 4 per cent. per annum = 1·02: then by Rule II. $1·02^4$ = 1·08243216: then 1·08243216 × £825 = 893·006532 = £893 0*s.* 1·56768*d.* *Ans.*

Reason of Rule II.—1°. The amount of £1 for a year is 1·05, and since any two principals must be proportional to their corresponding amounts, for the same time and at the same rate,

£1 *principal* : 1·05 *principal for second year* :: 1·05 *amount of* £1 *in a year* : 1·05 × 1·05 the amount of £1·05 in a year. Again, 1 : 1·05 :: 1·05 × 1·05 : 1·05 × 1·05 × 1·05, the amount of £1 for three years, which is the product of three factors, each factor being the amount of £1 for a year.

2°. It is evident that £1 : its amount for any time :: any other principal : its amount for the same time.

167. *To find the present worth of any sum to be paid hereafter at compound interest.*

Rule.—Divide the given sum by the amount of £1 for the given time and at the same rate.

Ex. 1. What sum will amount to £347 5*s.* 9*d.* in 3 years, at 5 per cent. compound interest?

The amount of £1, at 5 per cent., for 3 years is $1{\cdot}05^3 = 1{\cdot}157625$: then £347 5*s*. 9*d*. ÷ 1·157625 = £347·2875 ÷ 1·157625 = £300. ***Ans.***

Reason.—The reason is evident from **166.**

XLV.

1. What sum will produce £965 15*s*. 10¾*d*. in 4 years, at 6 per cent. per annum?

2. What sum will amount to £743 11*s*. 3½*d*. in 14 years, at 6 per cent. per annum?

3. What sum, in 3 years, will amount to £173 12*s*. 10½*d*., at 5 per ent. compound interest?

4. At 6 per cent. in 4 years, what will amount to £277 18*s*. 0·896*d*. t compound interest?

5. What sum, in one year, the interest accruing half-yearly, at 2½ per ent., will amount to £512 11*s*. 6¾*d*.?

India Board.

6. Find the amount of £4800 in 3 years, at 3⅛ per cent., compound terest?

Various Examinations.

7. What will £275 amount to in 4 years at 5 per cent., compound terest?

8. Find the amount of £240 in 4 years, at 3 per cent., compound terest.

9. Find the amount of £550 at the end of 4 years, at 5 per cent., mpound interest.

10. Find the amount of £8600 in 4 years, at 5½ per cent., compound terest.

11. Find the amount of £225 for 3 years, at 4¼ per cent., compound terest.

12. How much will £8000 amount to in 4 years, at 3¼ per cent., mpound interest?

LOGARITHMS.

168. A logarithm of a number 'is that index, which when onnected with some constant positive root, or base, represents the value of the number.' Logarithms serve to facilitate numerical computations. The base of our common system of logarithms is 10. Any other number may be made the base of a system of logarithms except unity; but 10 being the common ratio of our system of notation is peculiarly adapted for a base.

	Number.	Log.
10^1	10	1
10^2	100	2
10^3	1000	3
10^4	10000	4
10^5	100000	5
10^6	1000000	6

From the above table it is evident that, as the log of 10 is 1 and that of 100, 2, the logs of all numbers between 10 and 100 will be more than 1 and less than 2; the logs of all numbers between 100 and 1000 will be more than 2 and less than 3, and the logs of all numbers less than 10 will be less than 1.

The whole number 1, 2, 3, &c., which is always one less than the number of figures in a whole number, and in a decimal, one more than the number of prefixed ciphers, is called the *index* or *characteristic* of the logarithm, and being so easily known is never written in tables of logs. The decimal fraction annexed to the characteristic of a log, is called the *mantissa*. In fact, a log of a number is an index of the base expressing a certain power, to which if the base be involved it will produce that number.

If 6 be taken as the base of a system, we have:

	Number.	Log.
6^1	6	1
6^2	36	2
6^3	216	3
6^4	1296	4

and so on.

169. *To multiply numbers.*

Rule.—The number corresponding to the sum of their logs will be the product.

Thus to multiply 100 by 1000.

Log $100 + \log 1000 = 2 + 3 = 5$ the log of product, the number corresponding to which in the table is 100000.

170. *To divide numbers.*

Rule.—Subtract the log of the divisor from that of the dividend, and the number corresponding to the difference will be the quotient.

Thus to divide log 1000000 by 100.

$$\begin{array}{ll} \log 1000000 & = 6 \\ \log 100 & = \underline{2} \end{array}$$

log of quotient $= 4$, the number corresponding to which is 10000.

171. *To involve a number.*

Rule.—Multiply its log by the index of the power, and he product will be the log of the power.

Thus to raise 10 to the 5th power (10^5).

Log $10 = 1$: then $1 \times 5 = 5$, the number corresponding to which is 00000; hence $10^5 = 100000$.

172. *To evolve a number.*

Rule.—Divide its log by the index of the required root r log of root.

Thus, to find the $\sqrt{10000}$.

Log $10000 = 4$, and $4 \div 2$ (the index of the root) $= 2$, the number rresponding to which is 100; hence $\sqrt{10000} = 100$.

Find the $\sqrt[3]{1000000}$.

Log $1000000 = 6$, then $6 \div 3$ (the index of root) $= 2$, the number correonding to which is 100.

The above are the four fundamental rules in which the use of logs breviate calculations.

173. *Logs of the ten digits.*

Number. Log.
1 = 0
2 = ·3010300
3 = ·4771213
(2 × 2) 4 = ·6020600 = ·3010300 × 2
5 = ·6989700
(3 × 2) 6 = ·7781513 = ·4771213 × 2
7 = ·8450980
(4 × 2) 8 = ·9030900 = ·3010300 × 3, or = ·60206 + ·3010300
(3 × 3) 9 = ·9542425 = ·4771213 × 2
10 = 1·

Having the log of 2, we can find the logs of 4 (2 × 2), 8 (2 × 2 × 2), 16, and so on; and having log 3, we can find logs of 6, 9, 27, &c.; and also the logs of all numbers formed of these digits in same order, such as 40, 80, ·4, 04, &c. &c.

174. The *mantissa* of all numbers expressed by the same figures is the same, and the logs of such numbers will only vary in the *index* or *characteristic*; for any number will be divided by 10 by subtracting 1 from its characteristic.

Hence, log 9376·54 = 3·972043
log 937·654 = 2·972043
log 93·7654 = 1·972043
log 9·37654 = ·972043
log ·937654 = $\bar{1}$·972043
log ·0937654 = $\bar{2}$·972043

175. When the characteristic is negative, as that of a decimal, the negative sign is written over the number as above.

The log of a fraction will be the excess of the log of its numerator above that of its denominator. The log of a mixed number may be found by reducing it to an improper fraction.

176. *To find a fourth proportional to three numbers.*

Find a fourth proportional to 8 : 35 :: 36.

$$\begin{array}{rr} \log 35 = (\log 7 + \log 5) = & 1{\cdot}5440680 \\ \text{and } \log 36 = (\log 9 + \log 4) = & 1{\cdot}5563025 \\ \hline \text{sum of logs} & 3{\cdot}1003705 \\ \text{subtract } \log 8 = (\log 2 \times 3) = & {\cdot}9030900 \\ \hline \text{log of answer} = & 2{\cdot}1972805 \end{array}$$

The number corresponding to which is 157·5

The above result is found by adding the logs of the second and third terms, and subtracting log of first term from the sum.

Colonial Office.

Ex. 1. Define a logarithm, and having given log 2 = ·3010300, log 3 = ·4771213, find log 36, log 75, and log $\sqrt[6]{{\cdot}0012}$.

$$\text{Here, } \left.\begin{array}{r} \log 2 = {\cdot}3010300 \\ \log 3 = {\cdot}4771213 \end{array}\right\}\text{add}$$

$$\log 6 = {\cdot}7781513; \text{ hence } (\mathbf{171})$$

log 36 = 1·5563026.

Again, since log 6 = ·7781513 and log 10 = 1

log 60 is = 1·7781513, and since

log 2 = ·3010300, log 4 = ·6020600; therefore log 15 (60 ÷ 4) = 1·7781513 — ·6020600 = 1·1760913

$$\left.\begin{array}{r} \text{and } \log 150\ (15 \times 10) = 2{\cdot}1760913 \\ \log 2 = {\cdot}3010300 \end{array}\right\}\text{subtract}$$

$$\text{Hence } \log 75 (150 \div 2) = 1{\cdot}8750613$$

$$\left.\begin{array}{r} \text{Again, since } \log 6 = {\cdot}7781513 \\ \text{and } \log 2 = {\cdot}3010300 \end{array}\right\}\text{add}$$

$$\log 12 = 1{\cdot}0791813$$

hence log ·0012 = $\bar{3}{\cdot}0791813$; and log $\sqrt[6]{{\cdot}0012} = \bar{3}{\cdot}0791813 \div 6$* = $\bar{1}{\cdot}5131968$.

XLVI.

Admiralty.

1. What is the characteristic of a logarithm?

---

* In dividing a log with a negative characteristic by a whole number, add such a number as will make the characteristic divisible, thus: $\bar{3}{\cdot}7236390 \div 7 = (-4+4) + \bar{3}{\cdot}723628 \div 7 = \bar{7} + 4{\cdot}723628 \div 7 = \bar{1}{\cdot}674804$.

2. Having given log 2 = ·3010300, log 3 = ·4771213, find log 24, log 75, and $\sqrt[6]{\cdot 0144}$.

War Department.

3. Explain some of the uses of logarithms. Given log 2 = ·3010300, and log 3 = ·4771213, find the logarithms of 5, of 15, and of ·15.

India Board.

4. Find by logarithms the number of digits in $(512)^5$, and find also by logarithms a fourth proportional to ·00625, ·005, ·064, having given log 2 = ·301030.

Various Examinations.

5. What is a logarithm?

6. What is the ordinary base in use, and what is the peculiar advantage of it?

7.* Given log 3 = ·4771213, and log 7 = ·8450980, find log 63 and log of $\frac{1}{63}$.

8.* Find log of $5\frac{1}{3}$, given log 2 = ·301030 and log 3 = ·4771213.

EXAMINATION PAPERS.

XLVII.

The following 8 *questions have been set to Labourers, Viewers, &c., under War Office.*

1. Add together,	4275	2.	9036
	529		1572
	3708		29
	94		305

3. From	5923	4. From	6057
Take	3802	Take	3069

5. Multiply 235 by 3.
6. Multiply 567 by 13.
7. Divide 204 by 6.
8. Divide 493 by 23.

---

* Not proposed in any Department.

XLVIII.

The following 6 *questions were set to Warders, &c., in Prisons— Time allowed,* 20 *minutes.*

1. Add together,	5976	2.	9287
	176		5488
	5414		136
	483		8148
3.	2507	4.	187
	7447		2579
	284		8238
	4284		2254
5. From	75846	6. From	67833
Take	48588	Take	9934

XLIX.

Letter Carriers and Rural Messengers. (*Time* 15 *minutes.*)

1. Add together—

6021	2. 462	3. 8248	4. 524
656	3588	3207	9243
6728	306	589	3680
339	6834	2438	3054

L.

Set to Office Keepers, Messengers, &c. (*Time,* 2 *hours.*)

1. Write down in figures seventy thousand one hundred and three.
2. Write out in words 607035.
3. Add together—

		£	s.	d.
47961344	4.	1934	8	7½
78169405		1908	14	7¼
79818653		4398	10	9
96786637		8799	17	2½
35273337		817	3	6¾
2694153		4831	2	11½

5. From 206497677
   Take 166479499

6.		£	s.	d.
	From	30347	14	3½
	Take	28987	9	7¾

7. Multiply 4794601 by 304.
8. „ £9479 7*s.* 5½*d.* by 8.
9. „ £7321 2*s.* 9¾*d.* by 97.
10. Divide 2572347 by 7.
11. „ 2749344 by 56.
12. „ £2936 7*s.* 6*d.* by 6.

LI.

Out-door Officers in Customs. (Time, 3 hours.)

1. Write down under each other and add 367421, 5267, 924, 13 17890, 27, and 141109.

2. Subtract 5207 from 207202.

3. Add together—

3.
10099003
10584769
68537981
12659921
967989
2694135

4.	£	s.	d.
	3522	9	5
	304	12	3
	6763	9	10
	8713	8	10
	917	6	3
	4831	2	11

5. From 201076585
   Take 181589478

6.	£	s.	d.
	30608	14	2
	27876	16	7

7.	Tons	cwts.	qrs.	lbs.
	84	16	3	18
	1	2	2	14
	56	13	3	22
	19	8	2	16
	5	4	1	9

8.		Tons	cwts.	qrs.	lbs.
	From	54	8	1	17
	Take	9	16	3	22

9.		lbs.	oz.	dwts.	grs.
	From	169	6	14	17
	Take	83	7	15	20

10. Multiply 5301201 by 75023.
11. „ £4828 6*s.* 4¾*d.* by 7.
12. „ 5 tons 3 cwts. 2 qrs. 4 lbs. by 39.
13. „ £4516 11*s.* 6½*d.* by 620.
14. Divide 753477205876 by 5.
15. „ 679475247390 by 341.
16. „ 5 yds. 2 ft. 11 in. by 12.

17. Divide £3509768 16*s.* 1*d.* by 133.

18. In 49 tons 15 cwts. 2 qrs. 14 lbs. how many pounds?

19. In 467 tuns 3 hhds. 26 gallons 2 qrts. 1 pt. of wine how many pints?

20. In 2160000 cubic inches, how many loads of hewn timber?

21. Reduce 159 quarters of corn to quarts.

LII.

Open Competition for the situation of Apprentice in Dockyards.
(Time 3 hours.)

1. Write down in figures four hundred million five hundred and ten.

2. Write out in words 77005003.

3. Add together—

34567890
3729015
7684846
65478320
38140238
44596782

4. Add together—

£	*s.*	*d.*
2187	16	9
437	15	$6\frac{1}{4}$
456	19	$11\frac{3}{4}$
6391	4	1
3284	2	$1\frac{1}{4}$
59	12	$10\frac{3}{4}$

5. From 1974821862 take 820905820.

6. From £543061 12*s.* $2\frac{1}{2}$*d.* take £18921 15*s.* $5\frac{3}{4}$*d.*

7. Multiply 58640352 by 830506.

8. „ £24273 7*s.* $4\frac{1}{4}$*d.* by 9.

9. „ £7053 11*s.* $6\frac{1}{2}$*d.* by 52.

10. Divide 532659798203 by 467.

11. „ £4367074 1*s.* 0*d.* by 72.

12. „ £214673 3*s.* $3\frac{3}{4}$*d.* by 521.

13. Reduce 12 cwts. 3 qrs. 13 lbs. 5 oz. to ounces.

14. In 8709354 inches, how many miles, furlongs, poles, &c.?

15. Find the dividend on £7352 13*s.* 4*d.* at 12*s.* 6*d.* in the £.

16. If $8\frac{1}{4}$ cwts. cost 42 guineas, what is the price of $195\frac{1}{4}$ lbs.?

17. Add together $12\frac{2}{3}$, $2\frac{11}{18}$ and $\frac{23}{24}$.

18. Subtract $23\frac{4}{5}$ from $35\frac{7}{12}$.

19. Multiply $38\frac{1}{4}$ by $2\frac{2}{17}$.

20. Divide $11\frac{3}{5}$ by $2\frac{9}{10}$.

21. Add together 500·9, ·125, ·0087, and 903.

22. Subtract ·8309 from 407·02.

23. Multiply 1703·02 by 93·05.

24. Divide 112·03 by 98·52 to 3 places of decimals.

25. Divide 480 by ·000875.

26. Reduce 9*s.* 4½*d.* to the decimal of a pound.

27. Extract the cube root of 25 to four places of decimals.

28. Multiply by the method of duodecimals 3 ft. 1 in. 11 pts. by 4 ft. 6 in. 8 pts., and the product by 2 ft 7 in.

29. Express the result of the last question in cubic feet, cubic inches, and a fraction of a cubic inch.

30. Define 'Present Value,' and find the true discount on £501 10*s.* due six months hence, at 4 per cent. per annum, simple interest.

31. A bankrupt has good debts to the amount of £456 18*s.* 1*d.*, and the following bad debts, £360 7*s.* 10*d.*, £120 13*s.*, and £19 18*s.*, for which he receives respectively 4, 5, and 9 shillings in the pound; his own liabilities amount to £3408 12*s.*; how much can he pay in the £?

Twelve columns such as the following were given. (Time ½ an hour.)

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
32.	8139	7	4	33.	9065	13	8	34.	4123	17	6
	232	1	11		211	13	8		6904	7	3
	612	3	2		285	2	2		94	3	4
	9416	9	8		4321	18	9		8019	3	8
	1067	13	8		239	2	5		163	9	11
	207	4	6		2935	8	1		215	9	5
	8279	12	9		6409	12	8		9603	2	5
	752	15	3		3276	5	9		109	19	10
	287	6	8		210	14	8		6218	12	8
	1022	3	1		2015	1	2		1130	1	4
	1528	3	6		3925	4	11		6408	13	10
	46	3	6		631	7	7		1397	7	1
	1287	14	7		691	13	9		563	4	1
	3805	16	4		6127	9	10		4287	9	11
	4561	10	8		362	3	4		4201	6	8
	360	5	5		56	4	10		5976	2	5

In examinations which are not competitive six such columns as the above are given, but in competitive examinations twelve such as those at page 39 are given.

LIII.

The following paper was proposed where Elementary Arithmetic is to be known. (Time 2 hours.)

1. Write down in figures four hundred and one thousand three hundred and one.

2. Write down in figures two hundred millions eight thousand and eleven.

2. Write in words the number 90105003.

4. Add together—

10099003
10584769
86537981
12569921
969789
2694153

	£	s.	d.
5.	5322	9	5½
	304	12	3¾
	6736	9	10
	8713	8	10½
	917	3	6¾
	4831	2	11½

6. From 200175685
Take 181589478

	£	s.	d.
7.	30709	14	2½
	27876	16	7¼

8. Multiply 5301201 by 57023.
9. „ 429772 by 370010.
10. „ £4288 6 4¾ by 7.
11. „ £3509 1 2½ by 39.
12. „ £4156 11 6½ by 620.
13. Divide 573747025786 by 5.
14. „ 697457274930 by 341.
15. „ 107107648961 by 5037.
16. „ £2682076 2 6 by 12.
17. „ £3059678 17 1 by 133.

LIV.

Including Reduction, Rule of Three and Practice. (*Time* 1¼ *hour.*)

1. In 2306090 oz. how many tons, cwts., &c.

2. A ton of potatoes cost £7; what would have been the price of 24 lbs.?

3. Find (by Practice) the dividend on £1740 5*s.* at 14*s.* 2*d.* in the pound.

4. In 6 furlongs 4 poles 5 yards 2 feet, how many inches?

5. A man walks 17 miles 1650 yards in 3 hours 45 minutes; what is his rate per hour?

6. Find (by Practice) the price of 9 quarters 2 bushels 1 peck, at £2 16*s.* 8*d.* per quarter.

7. Reduce 600 half-guineas to half-crowns.

8. A bankrupt pays a dividend of 6*s.* 8*d.* in the pound; what is the loss of the creditor to whom he owes £750.

9. Find (by Practice) the rent of 23 acres 3 roods 15 perches, at £1 13*s.* 4*d.* per acre.

LV.

The following paper has been 'used in preliminary test examinations, and others which are not competitive.' * (*Time* 3 *hrs.*)

1. How many lbs. oz., &c. are there in 721572 grains of gold?

2. A ton of potatoes cost £7: how many pounds might have been bought for 3*s.*?

3. Find (by Practice) the dividend on £1430 12*s.* at 13*s.* 4*d.* in the pound.

4. Find the simple interest on £435 15*s.* for 2 years, at $2\frac{1}{2}$ per cent.

5. Add together $8\frac{1}{7}$, $\frac{9}{14}$, $4\frac{5}{21}$, and $2\frac{17}{42}$.

6. Subtract $3\frac{4}{45}$ from $8\frac{1}{30}$.

7. Multiply $9\frac{5}{11}$ by $4\frac{1}{8}$.

8. Divide $72\frac{1}{2}$ by $7\frac{1}{4}$.

9. Add together ·813, 420·91, ·00093, 7·043, and 12560.

10. Subtract 59·87 from 506·222.

11. Multiply 757·04 by 15·8.

12. Divide 22·097 by 543·14 to 3 places of decimals.

13. Find the value of 2·625 of a pound sterling.

14. In 5864542 inches, how many miles, furlongs, &c.?

15. If a horse trots $23\frac{3}{4}$ miles in 2 hours and a half, what is his rate per hour?

16. Find (by Practice) the price of 7 ozs. 14 dwts. 21 grs. of gold, at £2 13*s.* 4*d.* per ounce.

17. Find the amount of £2700 for 4 years at $3\frac{1}{2}$ per cent., compound interest (neglecting fractions of a penny).

18. Add together $12\frac{1}{3}$, $\frac{7}{15}$, $\frac{47}{20}$, and $\frac{7}{45}$.

19. Subtract $\frac{19}{52}$ from $4\frac{3}{26}$.

20. Multiply $18\frac{3}{4}$ by $1\frac{13}{15}$.

21. Divide $1\frac{7}{22}$ by $8\frac{2}{7}$.

22. Add together 50004, ·46, ·579, and 1201·043.

23. Subtract 19·9876 from 75.

24. Multiply 45·267 by 7·045.

---

* Where Vulgar and Decimal Fractions are required.

25. Divide 15·96 by ·065.

26. Reduce 18*s.* $8\frac{1}{4}$*d.* to a decimal of 2*s.* 2*d.*

27. Reduce 5 acres 13 poles to square feet.

28. If $4\frac{1}{8}$ cwts. of sugar cost 21 guineas, what is the price of $195\frac{1}{4}$ lbs.?

29. Find (by Practice) the price of 34 cubic yards 3 feet 288 inches of earth, at £3 6*s.* 9*d.* per yard.

30. In what time will £2220 be doubled at 6 per cent. simple interest?

31. Add together $\frac{7}{16}$, $6\frac{5}{48}$, $\frac{17}{6}$, and $\frac{13}{24}$.

32. Subtract $7\frac{15}{22}$ from $11\frac{3}{44}$.

33. Multiply together $\frac{19}{20}$, $11\frac{1}{9}$, and $3\frac{21}{38}$.

34. Divide $7\frac{7}{11}$ by $3\frac{3}{55}$.

35. Add together ·2764, 1824, 176·09032 and ·47.

36. Subtract 7·20147 from 872·70032.

37. Multiply 7·045 by 974·01.

38. Divide 1·765 by 2470 to five places of decimals.

39. Reduce 1*s.* 2*d.* to the decimal of 12*s.* $8\frac{1}{4}$*d.* to four places.

LVI.

The following paper was given in Competitive Examinations; Candidates not previously tested. (Time 3 hours.)

1. In 3 lbs. 10 ozs. 4 dwts. 12 grains, how many grains troy?

2. How many acres will 34 men reap in the time that 10 men take to reap 23 acres?

3. Find (by Practice) the price of 2 cwt. 1 qr. $10\frac{1}{2}$ lbs. of soap at 3*s.* 9*d.* per stone of 14 lbs.

4. Find the simple interest on £73 5*s.* 6*d.* for 5 years 6 months, at $3\frac{1}{2}$ per cent. per annum.

5. Add together $\frac{1}{2}$, $\frac{7}{12}$, $\frac{5}{6}$, and $\frac{4}{5}$.

6. Subtract $\frac{29}{60}$ from $1\frac{4}{15}$.

7. Multiply $\frac{21}{26}$ by $\frac{20}{51}$.

8. Divide $\frac{51}{120}$ by $\frac{7}{19}$.

9. Add together 802·573, 31·095, 9·7235, and ·507.

10. Subtract 11·709 from 12·0954.

11. Multiply 34·742 by 3·25.

12. Divide 169·24 by 549 to 3 places of decimals.

13. Find the value of 2·0875 of £1.

14. In 2771443 seconds, how many weeks, days, &c.?

15. A fortress is provisioned for 3 weeks at the rate of 15 ozs. a day for each man. If only $10\frac{1}{2}$ ozs. are served out daily to each man, how long can the place hold out?

16. Find (by Practice) the amount of a man's wages for 3 weeks 4 days 8 hours, at 6*s.* 8*d.* a day, reckoning 10 hours to a day.

17. Find the amount of £2700 for 4 years at 6 per cent. compound interest (neglecting fractions of a penny).

18. Add together $\frac{11}{12}$, $5\frac{13}{15}$, $\frac{21}{20}$, and $4\frac{5}{16}$.

19. Subtract $5\frac{11}{16}$ from $8\frac{11}{24}$.

20. Multiply $20\frac{4}{9}$ by $1\frac{13}{23}$.

21. Divide $2\frac{1}{12}$ by $4\frac{1}{4}$.

22. Add together ·024, 1000, ·00565, and 2·432.

23. Subtract ·099095 from 1·007.

24. Multiply ·51904 by 60·504.

25. Divide 24·1 by ·0025.

26. Reduce £2 11*s.* 6¾*d.* to the decimal of £1 10*s.*

[Six or seven *Additional Questions* were given here of more difficulty, such as Nos. 73, 74 and 75 in the 'Miscellaneous Questions' following these papers.]

LVII.

Proposed to Candidates for Inspectorships of Schools.

1. Pure gold is £3 17*s.* 10½*d.* per oz. and gold coin is composed of 21 parts of pure gold and 2 parts of alloy; the value of the alloy is 4½*d.* per oz.: required the value of alloy in 1000 sovereigns.

2. Multiply by the method of duodecimals 15 ft. 4 inches by 17 ft. 6 inches, and by 18 ft. 5 inches, and exhibit the result in cubic feet and a fraction of a cubic foot.

3. Find the amount of £1500 for a year at 5 per cent. per annum, compound interest, payable quarterly.

4. If £1000 be invested in the 5 per cents. at 97⅛, what interest has the invester for his money?

5. A grocer sold 1 lb. of tea and 3 lbs., of sugar for 6*s.*, but if the tea rose 10 per cent. and the sugar 50 per cent. they would have cost 7*s.* What was the price per lb. of each? *

6. A number of men were arranged into a hollow square 4 deep. In the outer rank there were 50 in the face of the square, and in the inner rank 44. How many men were there in all?

---

* From the question 1 lb. tea+3 lbs. sugar=6*s.*, and $1\frac{1}{10}$ lb. tea+4½ lbs. sugar=7*s.*: hence the difference $\frac{1}{10}$ lb. of tea+1½ lb. sugar=1*s.* or ⅕ lb. of tea+3 lbs. sugar=2*s.*

From 1 lb. tea+3 lbs. sugar=6*s.*
Take ⅕ „ +3 „ =2*s.*
⅘ lb. tea =4*s.*: whence 1 lb. tea=5*s.* and 1 lb. sugar=4*d.*

7. In how many years will £2500 double itself, allowing 15 per cent. per annum simple interest ? *

8. The sum of £1001 was laid out in the 3 per cents. at $89\frac{3}{8}$ and a whole year's dividend having been received upon it, it was sold out, the whole increase of capital being 72 guineas. Find at what price it was sold out.†

9. A sum is laid out in the 3 per cents. at $89\frac{3}{8}$ and a half-year's dividend received upon it; the stock being then sold at $94\frac{5}{8}$ and the whole increase of capital being £54, find the original sum laid out.‡

[Several other questions were given in the above paper of less difficulty. More difficult Examination Papers are given at page 202.]

MISCELLANEOUS QUESTIONS.

Ex. 1. 'At what time between 3 and 4 o'clock will the hands of a clock be together ?'

At 3, the minute hand points to 12 and the hour hand to 3. The minute hand travels 12 times as fast as the hour hand, and the question becomes 'In what time will the minute hand, travelling 12 minutes while the hour hand is going 1, overtake the other which has 15 minutes of a start ?'

It is evident 55 minutes will be gained in 60 by the minute hand.

∴ 11 minutes will be gained in 12 „

and 1 minute will be gained in $\frac{12}{11}=1\frac{1}{11}$ „

∴ 15 minutes will be gained in $(1\frac{1}{11}\times 15)$ $16\frac{4}{11}$. The time between 3 and 4 when the hands will be together is $16\frac{4}{11}$ minutes past 3.

Ex. 2. John can do a certain work in 12 hrs., James in 10 hrs., and Richard in 8 hrs.; in what time should they do it, all working together ?

---

* Any sum of money will double itself in the time found by dividing 100 by the rate.

† $89\frac{3}{8} : 100 :: £1001 : £1120$: then interest of £1120 at 3 per cent.=£33 12*s*., and £33 12*s*.÷2=£16 16*s*. the half-year's dividend; then £1001+72 guineas−£16 16*s*.= £1059 16*s*.; therefore $\frac{£1059\ 16s.\times 100}{1120}=94\frac{5}{8}$. *Ans.*

‡ Here, gain on every $89\frac{3}{8}$ invested$=94\frac{5}{8}-89\frac{3}{8}=5\frac{1}{4}$, to which add a half year's dividend, and $5\frac{1}{4}+1\frac{1}{2}=6\frac{3}{4}$ whole gain on £$89\frac{3}{8}$: whence $(54\div 6\frac{3}{4})$ $89\frac{3}{8}$=£715 *Ans.*

Here, John does $\frac{1}{6}$ in two hours.
James $\frac{1}{5}$ in two hours.
Richard $\frac{1}{4}$ in two hours.

All working together will do $\frac{1}{6}+\frac{1}{5}+\frac{1}{4}=\frac{37}{60}$ in two hours; therefore $\frac{1}{60}$ will be done in $\frac{1}{37}$ of 2 hrs. or $\frac{2}{37}$ of an hr.

And $\frac{60}{60}$, the whole, in $\frac{2 \times 60}{37}=\frac{120}{37}=3\frac{9}{37}$ hours. *Ans.*

Ex. 3. If oranges be bought at 10 for a penny, how many should be sold for 2*s.* 2*d.* to gain 30 per cent.?

To gain 30 per cent. 100 must be sold for the same money which 130 cost;

Now, cost of $10=1d.$
∴ Cost of $100=10d.$
Hence cost of $130=10d.+3d.=13d.$
Whence we have selling price of $100=13d.$

∴ Number which should be sold for 2*s.* 2*d.* $(13d. \times 2)=100 \times 2=200$. *Ans.*

Ex. 4. The annual losses of a merchant during 4 years average $3\frac{3}{4}$ per cent. upon his original capital; at the end of the four years his capital is £2783 15*s.*; what was it at first?

An average loss of $3\frac{3}{4}$ per cent. for 4 yrs. $=3\frac{3}{4} \times 4=15$. Whence an original capital of £100 would (in this time) be reduced to £85; whence, 85 : 100 :: £2783 15*s.* : £3275 *Ans.*

Ex. 5. If A do $\frac{3}{4}$ of a certain work in 2 hrs. and B $\frac{2}{3}$ of the remainder in 1 hour, and C finish it in 10 minutes: in what time will they do it all working together?

A will do $\frac{1}{2}$ of $\frac{3}{4}=\frac{3}{8}$ of it in an hour.
B will do $\frac{2}{3}$ of $\frac{1}{4}\ (1-\frac{3}{4})=\frac{1}{6}$ of it in an hour.
C will do $\frac{1}{3}$ of $\frac{1}{4}=\frac{1}{12}$ of it in 10 minutes;
hence, $\frac{1}{12} \times 6=\frac{1}{2}$ of it in an hour.

Therefore they will all do $\frac{3}{8}+\frac{1}{6}+\frac{1}{2}=\frac{23}{24}$ of it in an hour working together, and the whole work in $1\frac{1}{23}$ hours. *Ans.*

Ex. 6. If a man rows 14 miles in $3\frac{1}{2}$ hrs. against the stream, the rate of which is 6 miles an hour: in what time should he row 10 miles with the stream?

Against the stream he rows 14 miles in $3\frac{1}{2}$ hrs., that is, $14 \div 3\frac{1}{2}=4$ miles an hour; but the stream brings him back or resists him 6 miles an hour: hence his rate of rowing is 10 miles an hour. With the stream he will row $10 + 6 = 16$ miles an hour; therefore he will row 10 miles in $\frac{10 \times 1 \text{ hr.}}{16} = \frac{10}{16} = \frac{5}{8}$ of an hour $= 37$ m. 30 secs.

Ex. 7. The rate of a clock is $\cdot 0\dot{4}$ *per cent.* too fast; how much will it gain in a day?

It is evident that a day by the clock will be $\cdot 000\dot{4}$ ($\cdot 0\dot{4} \div 100$) too much; and $\cdot 000\dot{4} = \frac{4}{9000} = \frac{1}{2250}$ of a day is the time it will gain $=$ $\frac{1 \times \cancel{24} \times \cancel{60} \times \cancel{60}}{\cancel{2250}} = \frac{192}{5} = 38\frac{2}{5}$ secs. *Ans.*

Ex. 8. Simplify the expression

$$\frac{\frac{1}{3} \text{ of } \frac{4}{5} \div 2\frac{2}{5} + 2\frac{1}{2}}{\frac{1}{7} \text{ of } \frac{2}{5} + \frac{1}{2} \div \frac{4}{7} - \frac{3}{4}}$$

Taking the numerator we have $\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$, which $\div 2\frac{2}{5}$ gives $\frac{1}{9}$, to which add $2\frac{1}{2}$ and we have $2\frac{11}{18}$, the value of numerator. Again, $\frac{1}{7} \times \frac{2}{5} + \frac{1}{2} \div \frac{4}{7} - \frac{3}{4}$ $= \frac{2}{35} + \frac{1}{2} \div \frac{4}{7} - \frac{3}{4} = \frac{37}{70} \div \frac{4}{7} - \frac{3}{4} = \frac{37}{40} - \frac{3}{4} = \frac{7}{40}$, the value of denominator. Now $2\frac{11}{18} \div \frac{7}{40} = \frac{47}{18} \times \frac{40}{7} = 14\frac{58}{63}$. *Ans.*

LVIII.

Inland Revenue.

1. If 110 dollars are worth £23, what are 15 dollars 2 dimes 7 cents worth? (1 dol. = 10 dimes = 100 cents.)

2. Find (by Practice) the value of 5 acres 2 roods 11 perches, at £95 per acre.

3. Find the simple interest on £3728 19*s.* 7½*d.* for 4¾ years at 5 per cent.

4. Find the compound interest on £5750 for 3 years at 4¾ per cent.

5. Add together $9\frac{1}{8}$, $3\frac{5}{8}$, $2\frac{11}{12}$, and $\frac{5}{7}$; add $125\frac{2}{3}$, $2\frac{1}{6}$, $\frac{3}{5}$, $\frac{7}{9}$.

6. Add together 39·51, 425·76, ·237, 4·0032; and 52·11, 328·01, 64, ·024.

7. Reduce ⅝ of 13*s.* 6*d.* to the decimal of a guinea.

8. Find the value of ·38246 of 10*s.* 6*d.*

9. Extract the cube root of 589364291.

Clerks in Post Office, Metropolitan Police Courts, Department of Science and Art, Board of Trade, Fisheries (Scotland), and Prisons.

10. Find the difference between $\frac{3}{8}$ of $\frac{2}{9}$ of a mile, and $\frac{2}{9}$ of $\frac{1}{20}$ of $2\frac{6}{7}$ of a furlong.

11. Find the cost of carpeting a room that is 25 feet long and 20

feet wide, the price of a yard of carpet 2 feet 6 inches wide being 4*s.* 3*d.*

12. If when wheat is at 60*s.* a quarter the sixpenny loaf weighs 4 lbs. how much should be paid for 25 lbs. of bread when wheat is 40*s.* a quarter?

13. Divide ·121 by 11, by 110, and by ·0011.

14. Convert 22 lbs. 5 oz. 15 dwts. 20 grs. troy into avoirdupois weight; one lb. avoirdupois being to one pound troy as 175 to 144.

15. The shares of a given railway are at $44\frac{5}{8}$ when consols are at $70\frac{1}{8}$, what should be their price when consols are at $68\frac{3}{4}$?

16. If 27 cwts. 1 qr. 3 lbs. 8 oz. of sugar cost £87 6*s.* what will be the price of 7 lbs. ?

17. Express ·00325 as a vulgar fraction in its lowest terms.

18. Express 7*s.* $3\frac{3}{4}$*d.* as the decimal of a pound.

India Office.

19. If 56 cubic feet 1044 cubic inches of timber are required to floor a room 29 feet 3 inches by 25 feet 4 inches, what is the thickness of the boards?

20. A person sells out of the 3 per cent. consols at 99 and invests in Exchequer bills, bearing interest at the rate of $2\frac{1}{2}$*d.* a day per cent. when the bills are at a premium of 7*s.* 6*d.* What effect has this on his income?

21. In the month of December last the number of paupers in a certain union was 336, the number of women being double that of the men, and the children being as many as the men and women together. If a man cost one-third more than a woman, and three children as much as a man and a woman together, and the whole cost for the month be £83 6*s.*; how much is the daily cost of each man, woman, and child?

22. Multiply by the method of duodecimals 6 ft. 7 in. 5 parts by 8 ft. 3 in. 10 parts.

23. Express the result obtained in the last question in square feet, square inches, and a fraction of a square inch.

24. Find the cost of 75 cwts. 1 qr. 16 lbs. of sugar at £2 4*s.* 11*d.* per cwt.

25. A bankrupt owes £25962 10*s.*, what must his assets be worth in order that he may pay 7*s.* $11\frac{1}{2}$*d.* in the pound?

26. A tradesman's annual losses during 5 years average $1\frac{1}{4}$ per cent. on the capital with which he began, and at the end of the 5 years his effects are worth £2531 5*s.*, what capital did he begin with?

War Department.

27. If 90*s*. will pay 5 men for 12 days' work, how much will pay 32 men for 24 days' work?

28. What should the result be if the efficiency of the second set were half that of the first, and their day's work five-fourths as long?

29. Find the amount of £875 at the end of 15 months, at 4 per cent. per annum, compound interest, the interest accruing quarterly.

30. Add together $\frac{1}{8}$, $2\frac{1}{7}$, and $13\frac{3}{10}$; divide by $13\frac{1}{14}$, and subtract the result from $5\frac{3}{70}$.

31. Add together $\frac{13}{18}$ of a shilling, $\frac{11}{80}$ of half-a-crown, $\frac{2}{5}$ of a pound, and $\frac{1}{8}$ of a guinea. Express the result as the decimal of a pound.

32. Express ·01056 and $\cdot 158\dot{6}\dot{3}$ as vulgar fractions in their lowest terms.

33. Sum the arithmetical series $1 + 5 + 9 + \ldots$ to 21 terms.

Commissariat.

34. What would be the amount of a salary of £95 per annum, subject to a deduction of £2 7*s*. 6*d*. per cent. per annum, for 1 year and 25 days?

35. Suppose $6948\frac{5}{7}$ dollars were purchased at $48\frac{7}{8}$*d*. each, and that they were issued at $\frac{5}{9}$ of a penny more than that rate; what would be the total cost of the dollars, the total amount for which they were issued, and the total gain upon the transaction?

36. What will £100 6*s*. 8*d*. amount to with compound interest for five years at 5 per cent. per annum, the interest being paid annually?

Customs and Excise.

37. If 939 men consume 351 quarters of wheat in 168 days, how many will consume 1404 quarters in 56 days?

38. Silk sold at 12*s*. 6*d*. per yard gained £7 10*s*. per cent., what did it cost per yard?

India Office.

39. Reduce 2 furlongs 11 yards 1 foot 9 inches to the decimal of a mile.

Proposed to Candidates for Cadetships of Constabulary (Ireland).

40. Find the average of $16\frac{3}{4}$, $24\frac{1}{2}$, 12, $95\frac{5}{8}$, 0, 52, $4\frac{1}{8}$, and 1·625; express the fractional part decimally.

41. A merchant has teas worth 4*s*. 6*d*. and 3*s*. 6*d*. per lb. respectively, which he mixes in the proportion of 3 lbs. of the former to 2

of the latter, and sells the mixture at 4*s.* 4*d.* per lb. ; what does gain or lose per cent. ?

42. The populations of 3 towns in the year 1841 were 1587 64575, and 29185; and in the year 1851 the two former had increas respectively 4 and 8 per cent., and the latter had decreased 20 p cent. ; find the increase or decrease per cent. of the total population the three towns.

43. If the Roman Catholics are 3 to 1 of the population of Irela and the Protestant Dissenters bear the proportion of 2 to 3 to the mer bers of the Established Church; find the proportion (per cent.) whi the Protestant Dissenters bear to the Roman Catholics.

44. What must be the market value of 3 per cent. stock in ord that, after deducting the income-tax of 9*d.* in the £, it may yield 3½ p cent. interest ?

45. The cost price of a book is 12*s.* 6*d.* If the expense of sale 9 per cent. upon this, and the profit 21 per cent. upon the same, what the retail price ?

46. From the following table :—

Railway Passengers	Great Britain	France	Prussia
1st Class	4743213	2124917	307492
2nd Class	10291749	6172429	1292476
3rd Class	21409217	15473551	2374237

find the proportion per cent. which the whole number of 1st class pa sengers bears to the whole number of 2nd and 3rd class passengers.

47. The rate of a clock is ·075 per cent. too fast. How much w the clock gain in the week ?

Education Office (London).

48. If £3 = 20 thalers, 25 thalers = 93 francs, and 62 francs 25 gulden ; find how many gulden are equivalent to £1 sterling.

49. Reduce to its simplest form the expression :—

$$\frac{3}{5}+\frac{4}{3}+\frac{7}{6}+\frac{11}{30}+\frac{2}{45}-\frac{7}{18}.$$

50. Reduce to its simplest form the expression :—

$$\frac{3}{7} \text{ of } \frac{4\frac{5}{9}}{12\frac{1}{8}} \text{ of } \frac{3\frac{4}{11}}{11\frac{5}{7}} \div 1\frac{1}{11}.$$

51. Extract the square roots of 531441 and of $32\frac{1}{9}$.

52. Explain the nature and use of logarithms and the advantages of selecting the number 10 as the base of tabular logarithms.

53. Having given log 6·234 = ·794767 and log 6·235 = ·794836, find the log 623472 and the number whose logarithm is 2·794800.

54. A person's annual expenditure from the year 1833 to the year 1853 inclusive is £293 17*s.* 4*d.*; he finds that in 1833 he spent £283 17*s.* 10*d.*, and in 1854, £365 14*s.* 1*d.* What was the average annual expenditure from 1834 to 1854 inclusive?

55. Find the discount on £126 13*s.* 4*d* due 8 months hence at 4 per cent. simple interest. What would be the amount of the error in this case if interest were taken instead of discount, and in whose favour would the error be?

56. Find the amount of £13333 6*s.* 8*d.* for 4 years at 5 per cent. compound interest.

57. Divide 8740434091488 by 876.

58. The population of Great Britain in the year 1851 was 21121967 and the increase during the previous half century had been 93·5 per cent. What was the population in 1801?

Admiralty.

59. What is the value of:—

$$\frac{\frac{2}{5} \text{ of } 5\frac{3}{4}-\frac{3}{8} \text{ of } 4\frac{1}{4}}{\frac{7}{8} \text{ of } \frac{3}{7} \text{ of } 5\frac{1}{4}-\frac{5}{8} \text{ of } \frac{4}{15} \text{ of } 3\frac{2}{3}}$$

60. What decimal of a pound troy is 16 dwts.?

61. What sum placed out at interest at 4 per cent. would amount to £580 in 3 years, allowing simple interest?

62. What sum would amount to same sum in same time at compound interest?

Admiralty (Dockyards).

63. Find the value of $\cdot 0\dot{5}$ of $\cdot 0\dot{6}$ of a mile.

64. A merchant buys 15 dozen of port at 82*s.* per dozen, and 60 dozen more at 50*s.* per dozen; he mixes them and sells the mixture at 70 shillings per dozen; what profit per cent. does he realise on his original outlay?

65. Extract the cube root of 946966168.

Education Office (Dublin).

66. There are 58 insolvents, 3 of whom paid 20*s.* in the pound; 11, 12*s.* 6*d.*; 19, 7*s.* $1\frac{1}{2}$*d.*; 17, 3*s.* $4\frac{1}{2}$*d.*; 2, 1*s.* 6*d.*; the rest nothing. What was the average dividend?

67. If coffee worth 2*s.* 9*d.*, 3*s.* $2\frac{1}{2}$*d.*, and 2*s.* $4\frac{1}{2}$*d.* per lb., be mixed in equal quantities, and the mixture sold at 16*s.* 9*d.* per cwt., what is the gain or loss per cent. ?

68. If oranges be bought at the rate of 20 for a shilling, how many should be sold for £1 8*s.* to gain 40 per cent. ?

69. The number of persons employed in the railways are, respectively :—

Year	England		Scotland		Ireland	
		Miles		Miles		Miles
1852	55331	5433·9	8271	961·4	3999	680·4
1853	66267	7544·2	8979	977·9	5163	786·9

Hence find the average number of men per mile employed on railways in the whole United Kingdom, (1) in 1852, (2) 1853.

70. The price of the 3 per cents. and the market value of railway stock paying $5\frac{1}{4}$ per cent. being respectively $92\frac{5}{8}$ and $99\frac{1}{4}$, compare the rates of interest which investments in those stocks would give.

71. The rate of a clock is ·0625 too fast, how much fast would it go in a week ?

72. The populations of 3 towns in 1841 were 20565, 53415 and 30125, and in 1851 the two former increased respectively, 8 and 12 per cent., and the latter decreased 40 per cent.; find the increase or decrease per cent. of the total population of the 3 towns.

73. Find what will be the rate of interest in order that the discount on £1936 18*s.* at the end of 3 years may be £207 10*s.* 6*d.*

74. If 48 pioneers in 5 days of $12\frac{1}{2}$ hours dig a trench 139·75 yards long, $4\frac{1}{2}$ yards wide, and $2\frac{1}{2}$ yards deep; how many hours per day must 90 pioneers work, during 84 days, to dig a trench $4910\frac{1}{18}$ yards long, $4\frac{7}{8}$ yards wide, and $3\frac{1}{5}$ yards deep?

75. If by selling goods for £272 I lose 15 per cent., how much per cent. should I have lost or gained if I had sold them for 320 guineas?

Chief Secretary's Office (Ireland).

76. 100 lbs. bought @ 4*s.* 4*d.* per lb. were sold at 5*s.*, and 100 lbs. bought @ 6*d.* per lb. were sold at 7*d.*, find the total gain per cent.

77. Find the cube root of 84·9.

78. If £533 6*s.* 8*d.* be taken as payment of £560 due 2 years hence, calculate the rate of discount.

79. What must be the price of the 3 per cent. stock that I may gain 3¼ per cent. on my outlay after paying an income-tax of 10*d.* in the pound?

80. Multiply by duodecimals 3 feet 10 inches 11 parts, by 4 feet 7 inches and 3 parts, and by 1 foot 5 inches, and express the result in cubic feet and inches.

81. What is the length of one side of a square whose area is equal to a rectangle of 513 yards 1 foot 11 inches × 1628 yards 0 feet 11 inches?

82. Reduce to a simple fraction $\frac{3}{7} \times 1{\cdot}1\dot{6} + 2{\cdot}1\dot{5}\dot{6} \times \frac{11}{14}$.

83. *If* A. do ⅔ of a piece of work in 4 hours, and B. do ¾ of the remainder in 1 hour, and C. finish it in 20 minutes: in what time will they do it all working together?

84. If two clocks, one of which gains 2 minutes and the other loses 4 minutes a day, be set at noon, what hour will the second indicate when the first indicates noon 12 days afterwards?

Inland Revenue.

85. The imperial gallon contains 277·27 cubic inches, and a cubic foot of water weighs 62·42 lbs.: find the weight of a pint of water correctly to two places of decimals.

86. Extract the cube root of 731189187729.

87. Multiply by the method of duodecimals 2 feet 10 inches 11 parts by 3 feet 7 inches 5 parts; and the product by 1 foot 10 inches.

88. Express the result of the last question in cubic feet, cubic inches, and fraction of a cubic inch.

89. A sum of £3000 is to be divided between 20 men, 64 women, and 96 children: if each man's share is to be equal to the shares of two women, and if the 64 women are to have twice as much as the 96 children, how much will the several individuals receive?

90. When a vulgar fraction is reduced to a decimal, in what cases will the decimal terminate?

91. In what cases will it recur?

92. What is the limit in the number of figures in the period?

Constabulary Office.

93. Divide £177 7*s.* 0¾*d.* among four persons, in the proportion of the fractions ⅓, ¼, ⅕, ⅙.

94. Divide $4{\cdot}0\dot{3}$ by $\cdot1\dot{4}0\dot{7}$.

95. What is discount? If the discount on £567 be £34 14*s.* $3\frac{3}{7}$*d.*, simple interest being reckoned at $4\frac{1}{2}$ per cent. per annum, when is the sum due?

96. A. barters sugar with B. for flour which is worth 2*s.* 3*d.* per stone, but uses a false stone weight of $13\frac{1}{2}$ lbs. (instead of 14 lbs.), what value should B. set on his flour that the exchange may be fair?

97. A cwt. of potatoes cost 10*s.*, how many lbs. might have been bought for 1*s.* 8*d.*?

Education and Constabulary Offices.

98. The populations of three towns, in the year 1841, were 20325, 42405, and 1423; and in the year 1851 they had increased respectively 9, 10, and 12 per cent.; find the average population of the three towns in 1851.

99. If goods which were bought at £2 5*s.* 10*d.* per cwt. be sold at £2 14*s.* 1*d.*, what is the gain per cent.?

100. If by selling an article at 19*s.* 3*d.* a pound, I gain 12 per cent. on my outlay, what was its prime cost?

101. A grocer buys 3 cwts. of sugar at 5*d.* per lb., and 7 cwts. at $6\frac{1}{2}$*d.*: he sells $5\frac{1}{2}$ cwts. at $5\frac{1}{2}$*d.* per lb.; at what rate per lb. must he sell the remainder in order to make 15 per cent. on his outlay?

102. A person sells out of the 3 per cents. at 98, and invests his money in railway 5 per cent. stock at par; find how much per cent. his income is increased.

103. The present prices of the 3 per cent. consols and Midland Railway stock, paying $5\frac{1}{4}$ per cent., are respectively $95\frac{3}{8}$ and $108\frac{1}{2}$. Compare the rates of interest which investments in these stocks would give.

104. A person buys coffee at £5 12*s.* 6*d.* per cwt. and chicory at £2 5*s.* 5*d.*, and mixes them in the proportion of two of chicory to five of coffee. He retails the mixture at 1*s.* 3*d.* per lb. What is his gain per cent.?

105. By selling tea at 5*s.* 4*d.* per lb. a grocer clears $\frac{1}{8}$ of his outlay: he raises the price to 6*s.* 2*d.* per lb.: what does he clear per cent. upon his outlay at the latter price?

NOTE.—In all Examinations the time occupied by the candidate is punctually noted, and it is stated that 'additional credit will be given for rapidity.'

Set to all, except Tidewaiters, Weighers, Letter Carriers, and Messengers.

	£	s.	d.		£	s.	d.
106.	134	6	6	107.	8204	15	2½
	678	4	2		211	6	8
	1067	17	9½		7743	13	3¼
	4032	12	1		6917	11	9½
	9416	9	8½		14376	15	10¼
	1067	13	8		2371	11	9
	239	2	5¼		21309	5	8¾
	6409	12	8¼		1528	3	6
	752	15	3½		6408	13	10½
	6902	11	2		8411	1	5¼
	71203	2	4½		5298	4	9
	13257	8	11½		23198	2	7
	3276	5	9¼		631	7	7¼
	46	3	6¾		691	13	9½
	1287	14	7½		1414	12	4
	4917	10	8		5016	18	2
	90365	10	4		8756	14	9½
	360	5	5		56	4	10¾
	1379	17	2¾		616	3	7½
	9	9	10½		1189	2	3¾
	2963	4	5		401	3	6½
	906	10	7½		7116	15	7¾
	2222	5	1		917	15	8

Proposed to Candidates in various Departments, chiefly at Competitive Examinations.

108. At what rate per cent., simple interest, will £1245 amount to £2575 11*s.* 10½*d.* in 15 years?

109. What is the simple interest on £2245 for 5 years, at 4¾ per cent.?

110. Reduce to its simplest form the expression $\frac{1}{7}+\frac{3}{11}+\frac{17}{13}-\frac{581}{1001}$.

111. If 10 lbs. 3 oz. of tea at 4*s.* 8*d.* per pound be mixed with 23 lbs. 3 oz. at 5*s.* 4*d.* and 17 lbs. 11 oz. at 6*s.* 8*d.*, and the mixture be sold at 6*s.* 1*d.* per pound, what will be the gain or loss? and what the gain or loss per cent. on the cost price?

112. Reduce to its simplest form the expression :—

$\frac{2}{11}$ of $6\frac{10}{13}$ of $14\frac{3}{10}$, divided by $\frac{1}{17}$ of $299\frac{1}{5}$.

113. Find the value of $\frac{2}{7}$ of a pound $+\frac{47}{147}$ of a guinea $+\frac{211}{840}$ of 6*s.* 8*d.* $+\frac{5}{63}$ of a crown.

114. Find the interest on £1376 11*s*. 3*d*. at $4\frac{3}{4}$ per cent., lent on the 6th April, and repaid on the 29th of August, the former day being excluded and the latter included.

115. What sum of money put out at simple interest for 3 years, at $4\frac{1}{2}$ per cent. will amount to £1281 12*s*. 1*d*.

116. At what rate per cent. will the interest on £635 9*s*. $4\frac{1}{2}$*d*. amount to £38 2*s*. $6\frac{3}{4}$*d*. in two years?

117. At what rate per cent. will £651 13*s*. 4*d*. amount to £748 12*s*. $0\frac{1}{2}$*d*. in $3\frac{1}{2}$ years, at simple interest?

118. Reduce $\frac{36}{275}$ to a decimal showing the whole of the circulating period.

119. In what time will £2675 4*s*. 2*d*. amount to £3397 10*s*. $3\frac{1}{2}$*d*. at 6 per cent., simple interest?

120. A society subscribes £45 18*s*. 9*d*. to a charity; each member pays as many pence as there are members in the society; find the number of members.

121. If 10 men can mow 126 acres in 12 days of $9\frac{1}{2}$ hours each, how many can be mown by 45 men in 11 days of $8\frac{4}{5}$ hours each?

122. What is the difference between the interest and discount on £350 for 3 years at 4 per cent.?

123. Find the amount of £7000 in 4 years, at 5 per cent. per annum, compound interest.

124. Reduce ·06 of 4·2 of a guinea to the decimal of a pound.

125. Two persons have each a capital of £12000: the one invests it in the 3 per cent. consols at $90\frac{5}{8}$, the other in railway shares paying 5 per cent. at $103\frac{3}{4}$: find by how much the income of the one exceeds that of the other.

126. Reduce $\cdot 60\dot{1}24\dot{3}$ to a vulgar fraction in its lowest terms.

Various Departments.

127. Add together $\frac{1}{3} + \frac{2}{7} + \frac{5}{21} + 1\frac{2}{11} + \frac{8}{77}$ and subtract the result from $2\frac{1}{6}$.

128. What number added to $1\frac{7}{11}$, $2\frac{7}{18}$, $3\frac{5}{22}$, $\frac{9}{24}$, will make the sum total 10?

129. If $\frac{6}{7}$ of a quarter of wheat cost 54*s*. what will be the price of $\frac{4}{9}$ of a bushel?

130. Reduce $7\frac{1}{2}$ guineas to the decimal of £1000.

131. If 3 men can mow 14 acres of grass in 5 days of 9 hrs. each, in how many days of 10 hrs. each will 5 men mow 35 acres?

132. If by selling an article for £54 I lose 12 per cent., how much per cent. should I have lost or gained if I had sold it for 64 guineas?

133. Multiply by the method of duodecimals 7 feet 2 inches 3 parts by 6 feet 7 inches 5 parts, and the result by 2 inches 7 parts.

134. Express the answer to the last question in cubic measure.

135. Add $\frac{4}{7}$ of $\frac{3}{2}$ of $4\frac{5}{18}$ of a furlong to ·05 of ·0$\dot{6}$ of a mile.

136. The expense attending the production of a book the retail price of which is 7*s.* 6*d.* is 2*s.* 4½*d.* per copy. The publisher allows the bookseller 25 per cent. on the retail price, and gives 13 copies to the dozen; 3000 copies are printed and sold; the author is to have half the profits; how much will he receive?

137. Find the amount of £7000 in 4 years at 3 per cent. per annum, compound interest.

138. What is the simple interest on £652 14*s.* 2*d.* for 3½ years at 6 per cent.?

139. Reduce ·26$\dot{1}$5384$\dot{6}$ to a vulgar fraction in its lowest terms.

140. In what time will the interest on £75 12*s.* 6*d.* equal the interest on £15 2*s.* 6*d.* for 105 days at any rate of interest?

141. Supposing that in England gunpowder is made of 75 parts of nitre, 10 of sulphur, and 15 of charcoal; in France of 77 of nitre, 9 of sulphur, and 14 of charcoal: if a ton of each be mixed, what weight of nitre, sulphur, and charcoal will there be in the compound?

142. By a reduction of the interest on Exchequer Bills from 2½*d.* to 1½*d.* per cent. per day a person loses at the rate of £152 7*s.* 9*d.* per annum, what amount of Exchequer Bills does he hold?

143. How many yards of matting, 4·$\dot{8}$ feet broad, will cover a floor that is 27·$\dot{3}$ feet long and 20·1$\dot{6}$ feet broad?

144. If $\frac{2}{57}$ of $3\frac{1}{2}$ of $7\frac{3}{5}$ of $\frac{4}{7}$ of 36 lbs. of sugar cost 12*s.* 6½*d.* how much will 17 tons 17 cwts. cost?

145. Convert 44 lbs. 11 oz. 11 dwts. 16 grs. troy into avoirdupois weight, one pound avoirdupois being to one pound troy as 175 to 144.

146. If a coach travelling 9 miles an hour occupies 4 hours more than a railway train in a journey of 69 miles, what is the proportion between the rates of the coach and the train?

147. If A. invests £33075 in the purchase of 3 per cents. at 87½, and B. invests the same sum in the purchase of 4 per cents. at 94½, what will be the difference in their net incomes after deducting income-tax at 16*d.* in the pound?

148. What is the value of one pound avoirdupois in troy weight?

149. Find the fractions equivalent to the circulating decimals ·2$\dot{6}$ and ·83$\dot{2}\dot{6}$.

150. If oranges be bought at the rate of 20 for a shilling, how many should be sold for £2 8*s.* in order to gain 40 per cent. upon the outlay?

151. Find the simple interest on £291 13*s.* 4*d.* at 3¾ per cent. for 6 years.

152. Railway stock is sold for 108, and with the proceeds the government stock bought at $91\frac{1}{8}$; after a time the government stock is sold at $95\frac{3}{8}$ and the original stock repurchased at 109, leaving a profit of £109 on the transaction: find the amount of stock sold.*

153. What will be the cost of painting the walls of a room at 1*s.* 7*d.* per square yard, the length being 19 feet $10\frac{1}{4}$ inches, the breadth 16 feet $1\frac{3}{4}$ inches, and the height 10 feet 3 inches? Solve by decimals.

154. How much will £3500 amount to in 4 years at $4\frac{1}{2}$ per cent. compound interest?

155. At what rate per cent. will £1303 6*s.* 8*d.* amount to £1884 18*s.* 11*d.* in 7 years at simple interest?

156. If I buy 14 oxen for £157 5*s.* 10*d.* and sell 6 of them at £7 4*s.* each, for what must the remainder be sold that I may gain 4 per cent. on the whole?

157. Explain the difference between interest and discount, and find the present worth of £1215 due 4 years hence at $5\frac{3}{8}$ per cent.

158. If $1\frac{3}{7}$ of $\frac{3}{40}$ of $1\frac{3}{4}$ of a ton is worth £1 10*s.* what is the value of $\frac{3}{5}$ of it?

159. Extract the square root of $4\frac{213}{289}$.

160. Extract the cube root of ·51478848.

161. Given log 2 = ·3010300
and log 3 = ·4771213
find log $\frac{1}{96}$.

162. Divide 16 by ·2 and the result by ·0002.

163. Add together ·3211 of a guinea, $\frac{3}{8}$ of a crown, and ·6925 of a shilling, expressing the result as a decimal of a pound.

164. A besieged town containing 22,400 inhabitants has provisions for 3 weeks; how many must be sent away that they may be able to hold out 7 weeks?

165. If a rental of £8050 per annum be charged with a land-tax at £11 5*s.* per £100; what will be the net income?

166. Reduce $\cdot 1\dot{3}\dot{5}$ and $\cdot 012\dot{3}\dot{6}$ to vulgar fractions.

167. Divide 274·6 by 3·672 to 3 places of decimals.

168. If a 6*d.* loaf weighs 4·35 lbs. when wheat is 5·75*s.* per bushel, how much bread can be bought for 18·13*s.* when wheat is at 9·2*s.* per bushel?

169. Reduce $\cdot 39\dot{4}6385\dot{4}$ to a vulgar fraction in its lowest terms.

170. Add together ·004, 2·08, 314·6073, ·19784, 10000, 90·4; and subtract the result from 21118·0116.

---

* I cannot say, with *absolute certainty*, that this question was proposed at Civil Service Examinations.

171. Reduce $\cdot 1\dot{4}\dot{4}$ and $\cdot 34\dot{2}1\dot{8}$ to vulgar fractions.

172. Find the compound interest on £364 for 3 years at $3\frac{1}{4}$ per cent.

173. A person sells £7500 3 per cent. consols at $90\frac{5}{8}$ and invests it in railway shares paying 5 per cent. at $104\frac{3}{8}$; find how his income is affected, and the gain or loss per cent.

174. If by selling an article at 5*s.* 6*d.* I gain $\frac{3}{8}$ of my outlay, what would I have gained per cent. if I sold it at 6*s.* 6*d.*?

175. What sum of money will produce as much interest in $3\frac{1}{4}$ years as £210 3*s.* in 5 years 5 months?

176. Find the value of $\dfrac{\frac{2}{7} \text{ of } \frac{5}{11} \text{ of } 5\frac{1}{2} \div 2\frac{1}{3}}{\frac{4}{9} \text{ of } \frac{3}{13} \div 8\frac{2}{3}}$.

177. Divide the sum of $\frac{3}{5}$ of £3 7*s.* 6*d.* and $\frac{1}{6}$ of $4\frac{1}{2}$ guineas by $10\frac{5}{7}$.

178. If when wheat is 11*s.* 3*d.* the bushel the 6*d.* loaf weighs 24 oz., how much should it weigh when wheat is 84*s.* the quarter?

179. How many yards of paper 29 inches wide will paper a room which is 22 feet 9 inches long, 17 feet 3 inches wide, and 9 feet 8 inches high?

180. A bankrupt's effects amount to 2548 guineas, and his debts to £3057 12*s.*, what will his creditors receive in the £.

181. What must a person whose property is valued at £430 14*s.* contribute to a poor-rate of 10*d.* in the pound?

182. Add together $\frac{5}{6}$ of a pound, $\frac{7}{9}$ of £2 10*s.* 3*d.* and $3\frac{1}{3}$ of 14*s.*

183. Add together $\frac{3}{4}$, $\frac{2}{3}$, $\frac{5}{12}$, $\frac{7}{89}$, $\frac{11}{100}$, and subtract the result from $2\frac{1}{8}$.

184. Find the compound interest on £250 for two years at 4 per cent. per annum, the interest being payable every half year.

185. Reduce $\frac{3432}{2574}$ to its lowest terms and divide the result by $1\frac{1}{2}$.

186. Reduce $\frac{729}{12150}$ to a decimal.

187. Add together $\frac{6}{7}$ of a pound, $\frac{3}{8}$ of a guinea, $\frac{3}{560}$ of 6*s.* 8*d.*, and $\frac{1}{14}$ of half-a-crown.

188. The price of consols is $88\frac{3}{8}$, how many railway shares, the market price of which is £8 15*s.*, can be purchased with £1000 consols?

189. The rates of the express and mail trains on a railway are 40 and 28 miles per hour respectively: what time is saved by taking the faster train for a journey of 192 miles?

190. Reduce £5 16*s.* 4*d.* to a fraction of £16 8*s.* 8*d.*, expressing the result in its lowest terms.

191. If $\frac{7}{9}$ of an ounce troy be worth 5*s.* 10*d.*, what will be the price of $\frac{3}{10}$ of a pound troy?

192. Find the cost of papering a room $5\frac{1}{2}$ yards long, $3\frac{3}{4}$ yards broad, and 10 feet high, with paper 28 inches broad, at 6*s.* 9*d.* a dozen.

193. Define discount. If the discount on a bill of £226 2*s.* 8*d.*, due at the end of a year and a half, be £12 16*s.*, what is the rate of interest?

194. Divide ·015 by ·00005.

195. Divide 313·24 by 24997·5: showing the whole of the circulating period.

196. Express $\cdot 378\dot{2}1\dot{6}$ as a vulgar fraction in its lowest terms.

197. If I lay out £1911 in 3 per cent. consols at 79½, at what price must I sell out, to gain £150, paying ⅛ per cent. commission on each transaction?

198.* Extract the square root of 13104400.

199. Extract the cube root of 586376253.

200. Reduce ⅜ of 17*s.* 6*d.* to the decimal of a guinea.

201. Extract the cube root of 408518488.

202.† A railway cutting is 38 feet wide at bottom, and 74 feet at top, 35 feet deep, and ¾ of a mile long: how many solid yards of earth were removed?

203. A traveller sets out to walk from A. to B., at the rate of 4 miles an hour. One hour later another traveller sets out from B. towards A. at 3½ miles an hour: when they meet the first has walked 6 miles more than the second. Find the distance from A. to B.

204. How much will £500 amount to in 3 years, at 4⅛ per cent., compound interest?

205.‡ It is required to divide the number 34 into two parts such that the difference between the greater and 18 shall be to the difference between 18 and the less in the ratio of 2 to 3.

206.§ A man rows 10 miles in 2½ hours against the stream, the rate of which is 3 miles an hour: how long will he be in rowing 5 miles with the stream?

Board of Works (Ireland).

207. A ship 40 miles from shore springs a leak which admits 3¾ tons of water in 12 minutes; 60 tons would be sufficient to sink her, but the ship's pumps can throw out 12 tons of water in an hour. Find her average rate of sailing that she may reach the shore just as she is beginning to sink.

208. A solar day being 24 solar hours, a sidereal day is 23 hours,

---

* Proposed in Admiralty Dockyards.

† Proposed in Education Office, Dublin.

‡ Proposed in War Department.

§ Proposed in Education Office, Dublin.

56 minutes, 4 seconds $\frac{93}{100}$. A sidereal day being now divided into 24 sidereal hours, find the number of sidereal hours, minutes, and seconds in a solar day.

209. What must be the market value of 4 per cent. stock, in order that, after deducting the income-tax of 10*d.* in the pound, it may yield 4½ per cent. interest?

210. Find the cost of laying with gold a table $6{\cdot}4\dot{2}8571\dot{4}$ yards long and $1{\cdot}7\dot{8}$ yards wide, at 25*s.* the square inch.

211. If 24 pioneers in 2½ days of 12½ hours long can dig a trench 139·75 yards long, 4½ yards wide, and 2½ yards deep: how many hours per day must 180 pioneers work during 21 days in order to dig a trench 4910$\frac{1}{16}$ yards long, 4⅞ yards wide, and 3⅕ yards deep?

212. Borrowed £220 at 5 per cent., and 12 months after £340 at 4 per cent.: when paid it took £607 18*s.* to discharge them both: how long had these sums been kept?

213. A. had 12 pipes of wine which he sold to B. at 4½ per cent. profit, B. sold them to C. for £40 12*s.* profit, C. sold them to D. for £605 18*s.*, who cleared 6 per cent: what did A. give a gallon for the wine?

214. What will be the expense of flooring 50 feet 6 inches long by 32 feet 9 inches broad at £3 15*s.* per 100 square feet?

215. A person has ⅗ of a ship, he sells ⅔ of his share for £1260, what was the value of the ship?

216. If by remitting to Holland, at 34*s.* 6*d.* per £1, 4½ per cent. is gained, what is the rate of exchange when 10 per cent. is gained?

217. A person sold goods for 60 guineas, and lost 17 per cent., how much under their value were they sold?

Various Departments.

218. Bought a quantity of goods for £375, and 3 months after sold them for £412 10*s.*, how much per cent. per annum was the gain?

219.* The content of a cistern is the sum of two cubes, whose edges are 10 inches and 2 inches, and the area of its base is the difference of two squares whose sides are 1$\frac{1}{9}$ and 1$\frac{2}{9}$ feet. Find its depth.

220. How much paper, ¾ yard wide, will be required for a room that is 22 feet long, 14 feet wide, and 9 feet high, if there be three windows and two doors, each 6 feet by 3 feet?

221. A stone 18 inches long, 17 broad, and 7 deep, weighs 278 lbs.: how many cubic feet of this kind of stone will freight a vessel of 230 tons burthen?

---

* Proposed to candidates for a clerkship in Education Office, Dublin, July 1862.

Constabulary Office.

222. Extract the fourth root of 3841600.

223. Extract the fourth root of 752953600.

EXAMINATION PAPERS.

A Paper used in Competitive Examinations (Candidates previously tested.)

LIX.

Averages and Per-centages. (*Time 2 hours.*)

1. Find the average of $13\frac{12}{25}$, 21, $7\frac{3}{4}$, ·0023, $3\frac{1}{8}$, 0, $106\frac{1}{2}$, and $57\frac{7}{20}$: express the fractional part decimally.

2. If by selling wine at 15*s.* per gallon I lose 6 per cent., at what price must I sell it to gain $17\frac{1}{2}$ per cent.?

3. Of 32 selected candidates for the East Indian Civil Service in 1859, 3 were above 20 years of age when they went to India, 4 above 21, 12 above 22 and 23 respectively, and 1 above 24. From these data, find what is the average age at which the men went to India.

4. A merchant has teas worth 4*s.* 6*d.* and 3*s.* 6*d.* per lb. respectively, which he mixes in the proportion of 3 lbs. of the former to 2 of the latter, and sells the mixture at 4*s.* 4*d.* per lb.: what does he gain or lose per cent.?

5. Between the years 1841 and 1851 the population of England increased 14·2 per cent. In the latter year it was 21121290. What was it in the former year?

6. A person invests £5460 in the 3 per cents. at 91, he sells out 2000 stock when they have risen to $93\frac{1}{2}$, and the remainder when they have fallen to 85: he then invests the produce in the $4\frac{1}{2}$ per cents. at 102. What is the difference in his income?

7. What must be the market value of 6 per cent. stock, in order that, after deducting the income-tax of 10*d.* in the pound, it may yield $6\frac{1}{2}$ per cent. interest.

8. This was the same as No. 43 in the preceding 'Miscellaneous Questions.'

This Paper was also proposed in Competitive Examinations.
(Time 2 hours.)

LX.

1. When a 3¾ per cent. stock is at 93, find what price a 4½ per cent. stock must bear, that an investment may be made with equal advantage in either stock.

2. A person sells Midland Stock paying 6½ per cent. at 128½, and invests in Great Western stock paying 3 per cent. at 72½. By how much per cent. will the interest of his investment be altered?

3. A person invests £5000 in the new 6 per cent. Turkish loan, issued at 68 per cent. at 2½ premium. How much stock will he have, and what rate of interest will the investment give?

4. What must be the market value of 3 per cent. stock, in order that, after deducting an income-tax of 10*d.* in the pound, it may yield 3½ per cent. interest?

5. What is meant by the *par of exchange* between two countries? When is the exchange said to be against a country? Explain briefly why the course of exchange between two countries varies.*

6. If £3 = 20 thalers; 25 thalers = 93 francs; 27 francs = 5 scudi; and 62 scudi = 135 gulden: how many gulden = £1?

7. A trader in London owes a debt of 1000 pistoles to one in Cadiz: find what he gains by sending it to him through France, the exchanges being £1 = 25·4 francs; 19 francs = 1 Spanish pistole; 4 Spanish pistoles = £3.

8. A person in London owes another in St. Petersburg 920 roubles, which must be remitted through Paris. He pays the requisite sum to his broker, at a time when the exchange between London and Paris is 25·15 francs for £1, and between Paris and St. Petersburg 1·2 francs for 1 rouble. The remittance is delayed until the rates are 25·35 francs for £1, and 1·15 francs for 1 rouble. What does the broker gain or lose by the delay?

---

* 'The *par of exchange* is the quantity of money of one country intrinsically equal to a certain quantity of money of another.' Exchange is against a country when a given sum in that country's currency will purchase fewer bills than usual of another country. When the demand for bills upon any country is great from commercial causes, the course of exchange will vary, and the price of the bills will advance: and *vice versa*. 'When any cause makes the imports from a country greater than the exports to it,' the course of exchange will vary.

The following 15 *Questions were proposed to Candidates who had already passed a Test Examination.* (*Time* 3 *hours.*)

LXI.

1. If a steamer makes the passage from New York to Liverpool (say 2,760 miles) in 9 days 14 hours, and a train goes from London to Edinburgh (say 405 miles) in 18 hours: compare the rates of the steamer and the train.

2. Find the square root exactly of $2515\frac{294}{361}$.

3. Extract the cube root of 5·78 to three places of decimals.

4. Multiply by the method of duodecimals 3 feet 1 inch 11 parts by 2 feet 6 inches 7 parts, and the product by 1 feet 7 inches.

5. Express the result of the last question in cubic feet, cubic inches, and a fraction of a cubic inch.

6. Divide $4{\cdot}\dot{0}\dot{3}$ by ${\cdot}1\dot{4}0\dot{7}$.

7. Find the average of $21\frac{2}{3}$, $73\frac{4}{5}$, 0, 3·065, 82, $17\frac{3}{20}$, $5\frac{1}{4}$, $9\frac{5}{12}$: express the fractional part decimally.

8. A person sells as many 3 per cent. consols at $98\frac{5}{8}$ as produce £2000, and invests this sum in railway stock, paying $4\frac{1}{4}$ per cent. at $93\frac{3}{4}$. How is his income affected?

9. A person buys coffee at £5 12*s.* 6*d.* per cwt., and chicory at £2 5*s.* 5*d.* per cwt. and mixes them in the proportion of 2 of chicory to 5 of coffee. He retails the mixture at 1*s.* 3*d.* per lb. What is his gain per cent.?

10. Find the true discount on £512 15*s.* 3*d.*, due 52 days hence, at $2\frac{1}{2}$*d.* per cent. a day.

11. If 5 men can perform a piece of work in 12 days of 10 hours each, how many men will perform a piece of work four times as large in a fifth part of the time, if they work the same number of hours in the day, supposing that 2 of the second set can do as much work in an hour as three of the first set.

12. A canal 10 miles long is 8 yards wide at the top, 6 yards wide at the bottom, and 5 feet deep. How soon would the excavation of it be completed by 800 men, each removing on an average 15 cubic yards per day?

13. The rate of a clock is ·0375 *per cent.* too fast. How much will the clock gain in a week?

14. A vessel whose speed was $9\frac{1}{4}$ miles per hour started at 8 o'clock to go a distance of 74 miles. A second vessel, whose speed was to that of the first as 8 to 5, starting from the same place, arrived 5 minutes before the first. When did the second vessel start?

15. At a siege it was found that a certain length of trench could be dug by the soldiers and navvies in 4 days, but that when only half the navvies were present, it required 7 days to dig the same length of trench. What proportion of the work was done by the soldiers?

LXII.

London University.

1. Find the least number divisible by all the numbers from 1 to 12.

2. Simplify the expression:

$$\frac{4\frac{1}{2}-3\frac{1}{3}+5\frac{1}{12}}{7\frac{1}{2}-4\frac{1}{3}+11\frac{1}{12}}-\frac{11\frac{3}{4}-5\frac{7}{15}}{11\frac{3}{4}+5\frac{7}{15}}.$$

3. Extract the square roots of 443556; ·0000004489; and ·4.

4. Divide ·000356 by ·0000057, and calculate to 5 places of decimals the value of $\frac{1}{\sqrt{2}}$

5. Define Interest, Discount, and Present Value, and find the present value of £911 13*s.* 3*d.* due 5 years hence at 3 per cent. simple interest.

6. If the 3½ per cents. be at £95⅝ and the 3 per cents. at 82, which is the better investment?

7. A person walking along the road in a fog meets one waggon and overtakes another which is travelling at the same rate as the former. and he observes that between the time of his first seeing and passing the waggons, he walks 20 yds. and 60 yds. respectively; find how far he can see in the fog, and compare his rate of walking with the rate at which each waggon is moving?

8. The weights of pure gold and of alloy in a sovereign are very nearly in the ratio of 113 : 10 and the price of pure gold is £3 17*s.* 10½*d.* per oz.; find the weight of a sovereign, supposing its value to be that of the pure gold it contains.

9. Find the square root of $\cdot\dot{0}12345678\dot{9}$.

10. How many yards of carpet, three quarters of a yard wide, will cover a room whose width is 20 ft. and length 22 ft.?

11. By selling tea at 4*s.* 6*d.* per lb. a grocer cleared $\frac{1}{11}$ of his outlay, what would he have gained if he had sold the tea at 5*s.*?

12. Express $\frac{2}{9}$ of 7*s.* 6*d.* + ·625 of 16*s.* as a decimal of £2 1*s.* 8*d.* Divide 2·88 by ·00016.

13. Find the square of $\cdot\dot{2}$ in the form of a recurring decimal; and find $\sqrt{\cdot 69\dot{4}}$.

14. An electric cable is laid between two ports and it is found that the gross annual receipts amount to 20 per cent. on the original outlay;

the annual cost of repairs is 2 per cent. on the original cost, and the expense of working is found to be £600 a year, in addition to 20 per cent. on the *net* receipts. After a year a second cable is laid down, and it is found that the gross receipts and the expense of working are doubled, while the cost of repairs for the new cable is only 1 per cent., and that the *net* receipts now amount to £14500. Find the cost of a cable and the *net* receipts for the first year.

15. A person invests in £10 railway shares when they are at a premium of 10*s*. At the end of a year he receives a guinea per share: what interest does he get?

16. If 81 bushels of wheat are consumed by 56 men in 5 days, how long will 16 men take to consume 28 bushels?

17. Having given log 2 = ·301030, and log 3 = ·477121, find log ·0144.

18. Find the characteristic of log 3·2 to base 5.

Chelsea Hospital—Woolwich.

19. A legacy of £897 1*s*. is to be divided among 3 persons A, B, C. A is to receive $\frac{1}{7}$; B is to receive $\frac{2}{3}$, and C the remainder. Find the sum to be received by each, and the fraction of the whole paid to C.

20. In what number of years will £1876 10*s*. amount to £2439 9*s*. at 4 per cent. simple interest?

21. A wine merchant mixes two kinds of wine and sells the mixture so as to gain 8 per cent. on what the wine cost him. Had he sold each kind of wine at the same price per gallon as he sells the mixture, he would have gained 10 per cent. and 6 per cent. respectively on the cost price of each. In what proportion were the two kinds of wine mixed?

22. The stuff out of a lead mine contains at first 15·9 per cent. of lead. After washing, by which process the amount of lead ore is not diminished, the stuff contains 87·45 per cent. of lead. How much rock was washed away out of 216 tons 5 cwts. of the original stuff?

23. The income derived by a legatee from money invested in his behalf in the 3 per cents. at $93\frac{1}{2}$ is £68 3*s*. 6*d*. What was the amount of the legacy?

Direct Commissions—January 1863.

24. In standard gold 11 parts of 12 are pure gold. What weight of alloy is there in 3 oz. 5 dwts. of standard gold?

25. Add $\frac{5}{7}$, $\frac{1}{2}$, $\frac{1}{8}$, and $\frac{3}{28}$ of $3\frac{1}{3}$.

26. Find the value of ·36875 of a sovereign.

27. Divide 27·5264 by ·0374.

28. In how many days will the interest on £4600 at $1\frac{1}{2}$*d.* per cent. daily amount to £18 8*s.*?

29. If the corn of 13 horses for 63 days cost £17 6*s.* 8*d.* when corn is 4*s.* per bushel, how many horses will cost £10 13*s.* 4*d.* for corn in 56 days when corn is 4*s.* 6*d.* per bushel?

30. Extract the square roots of 8 and ·0002 each to three places of decimals, and multiply the square roots together.

Woolwich, 1861.

31. A sovereign standard gold weighs 5·136 dwts.; a shilling standard silver weighs $\frac{1}{66}$ of a lb. Troy; what weight in standard silver is equal to 4 oz. standard gold?

32. If a merchant with a capital of £20000 gain £500 in months, what sum will he gain with a capital of £30000 in 7 months?

33. Add together $\frac{5}{18}$ of a guinea, $\frac{3}{32}$ of a pound, $\frac{7}{20}$ of a crown, and $\frac{5}{18}$ of a shilling, and reduce the result to the decimal of a pound.

Queen's University—Ireland (Externs).

34. Reduce to simple fractions:

$$\tfrac{2}{3}+\tfrac{3}{4}+\tfrac{1}{12},\ \tfrac{3}{5}+\tfrac{1}{2}-\tfrac{7}{10},\ \text{and}\ \tfrac{2}{3}\times\tfrac{9}{10}\times 4\tfrac{1}{4}.$$

35. Divide $\frac{3}{4}$ of $2\frac{1}{3}$ by $\frac{5}{6}$ of $7\frac{1}{2}$.

36. Find the simple interest of £275 10*s.* for 2 years 146 days, at 5 per cent.

37. Extract the square root of 1562500 and of ·015625.

38. Multiply $3\frac{1}{2}$ by $4\frac{1}{4}$ and from the result subtract $7\frac{5}{8}$.

Woolwich and Chelsea.

39. Find without the use of the tables the number of which ·3 is the logarithm to a base 10.

Woolwich.

40. In the 3 per cents. what fraction of a given amount of stock is paid for annual interest, (1) without any deduction; (2) after 9*d.* in the pound has been deducted for income-tax?

41. If an oz. of standard gold, of which the weight of the alloy is represented by 2 parts out of 12, be worth £3 17*s.* 6*d.*, what is the value of 10 lbs. of jeweller's gold in which the weight of the alloy is represented by the decimal $\cdot 41\dot{6}$, the value of the alloy being neglected?

42. What is the base of the system of logs in which log 10 = 2?

43. Find the amount of £680 10*s.* in 20 yrs. at $3\frac{1}{2}$ per cent. per annum, compound interest, and payable half-yearly.

44. Find the side of a square courtyard the expense of paving which at 3*s*. 9*d*. per square yard is £38 10*s*. 5*d*.?

45. If 21 horses and 217 sheep can be kept 10 days for £56 8*s*. 4*d*., what sum will keep 9 horses and 60 sheep for 27 days, supposing that 3 horses eat as much as 50 sheep?

46. A manufacturer having a capital of £5000 on which he can realise by hand labour 10 per cent. profit, buys a machine for £1000, by which his profit on the remainder of his capital is raised to 20 per cent. This machine lasts 5 years. Of how much is he by that time a gainer by the machine, supposing him to draw £300 a year for the support of his family, and to reinvest annually the remainder of his profit in the business.

47. A dealer having laid in a stock of a certain article began to sell it by retail. The first day he made a profit of 3*d*., the second day of 4·2*d*. and so on, the profit increasing by 1·2*d*. a day, until the whole stock was disposed of; he then found that he had realised a profit of 14*s*. 3*d*. For how many days did he continue to sell the article?

Chelsea.

48. A person paid £18 15*s*. for a year's income-tax, but after government raised the tax by 9*d*. in the pound he paid £52 10*s*. What was the person's income, and at what rate in the pound was the tax levied at first?

49. A merchant made a mixture of wine at 28*s*. a gallon with brandy at 42*s*. a gallon, and he found that by selling the mixture at 35*s*. a gallon he gained 15 per cent. on the price of the wine, and 20 per cent. on the price of the brandy: in what ratio were the wine and brandy mixed together?

Direct Commissions.

50. Two shepherds A and B owning a flock of sheep agree to divide it. A takes 144 sheep and B takes 184 sheep and pays £70 to A; required the value of a sheep.

Miscellaneous.

51. A person after paying 10*d*. in the £ as income-tax, has remaining an income of £862 10*s*.: what would have been his income had there been no income-tax?

52. If the French 3 per cents. are at 60 when the English are at 95, the exchange between the countries being 25 francs to the pound, how much French stock in francs can be bought by selling £6000 stock out of the English funds?

ANSWERS.

I.

1. 590
2. 1403.
3. 2077.
4. 4570.
5. 8009.
6. 9097.
7. 461001.
8. 2005087.
9. 27000706.
10. 840020.
11. 10004008.
12. 1102070050.
13. 90040.
14. 304040004.
15. 603050.
16. 7003040.
17. 8000001002.
18. 500401.
19. 30007040.
20. 506000020.
21. 4000505.
22. 8000000600.
23. 10020001.
24. 400004040.

II.

1. 218721.
2. 3634130.
3. 38203510748.
4. 2159758.
5. 2073488.
6. 1143459.
7. 1423010.
8. 1608991.
9. 3292.
10. 1967324.
11. 176888441.
12. 12413.
13. 304125276.
14. 305852878.
15. 567036010.
16. 504605285
17. 280575843.
18. 485586.
19. 246776832.
20. 303225276.
21. 6027.
22. 3401.
23. 6486.
24. 4825.

III.

1. 61333.
2. 66641.
3. 217049.
4. 21107.
5. 96444.
6. 2165.
7. 92514.
8. 662733.
9. 47693.
10. 1618.
11. 6779495701.
12. 593684253.
13. 4228908880.
14. 4230399880.
15. 27909498.
16. 163161134.

IV.

1. 690739.
2. 702621.
3. 947089.
4. 1171740.
5. 566952.
6. 8690693.
7. 199617 : 243012.
8. 3694992 : 4310824.
9. 7278538 : 8475010.
10. 81745410 : 87768756.
11. 109344206 : 116232030.
12. 54949721 : 62052737.
13. 43606640 : 76155882.
14. 68604840 : 72808200.
15. 9003456000 : 851576880.
16. 3996915833 : 80737410.
17. 4342886402400.
18. 246204228753000.
19. 97152093486360.
20. 49631586380800.
21. 9108568931220.
22. 751680000 : 146880000.
23. 151240000 : 129350000.
24. 5551104 : 6244992.
25. 10611337 : 11576004.
26. 5024659561.
27. 46129276014848.
28. 5690578298.
29. 51734631391872.
30. 3592212.
31. 1486958060.
32. 3131672.
33. 1000362150.
34. 250075455045.
35. 69162164100652.
36. 9523916224416.

V.

1. 32885 + 1 : 19731 + 1.
2. 11899 + 2 : 10411 + 7.
3. 1038816 : 779112.
4. 3071 : 2943 + 1.
5. 3338 : 2616 + 10.
6. 2966 + 21 : 2559 + 72.
7. 9159 + 26 : 8468.
8. 15423 + 148 : 12325 + 65.
9. 11878 : 11136 + 508.
10. 953 + 368 : 917 + 8.
11. 932746.
12. 829107 + 5.
13. 62084251446 + 4.
14. 8866443377.
15. 120874856968.
16. 967427210 + 61.
17. 908382685 + 366.
18. 25484 + 44.
19. 457878404 + 311.
20. 12086960 + 59.
21. 30000 + 56.
22. 55655242 + 37.
23. 11593 : 6011 + 15.
24. 7282 : 6499 + 77.
25. 14556 : 116751 + 12.
26. 12247 + 441 : 161969 + 16.
27. 10979 + 319 : 80515 + 67.
28. 488 + 1179 : 684 + 379.
29. 4697 + 86 : 74 + 3586.
30. 5800 + 400 : 5301 + 540.
31. 38745 + 20 : 27675 + 20.
32. 14964 + 55 : 7183 + 20.
33. 39147 + 10 : 21748 + 25.
34. 39873 + 21 : 22152 + 6.
35. 1904916 : 3565612.
36. 3996370 : 7495894.
37. 9155 + 3½ : 3410 + 21.
38. 11805 + ½ : 2724 + 7¼.

VI.

1. 1452507303.
2. 117288146.
3. 1527 + 7.
4. 74813.
5. 788910.
6. 13707360.
7. 102901.
8. 999.
9. 10670 : 8536 : 10246401 : 9
10. 1067.
11. 23887872.
12. 17030.
13. 152 days.
14. 2397 days.
15. £14400.
16. 66.
17. 477120.
18. 385*d.*
19. 11.
20. 14*s.* and 11*s.*
21. 30.
22. 1038832.
23. 3613.
24. 4620.
25. 145881529087.

VII.

1. 15120*f.*
2. 81120 feet.
3. 76886 oz.
4. 42281 lbs.
5. 2703294 in.
6. 124614 oz.
7. 44387 oz.
8. 582205 oz.
9. 865170 in.
10. 2689542 in.
11. 40704 qrts.
12. 200944 grs.
13. 136240 sq. ft.
14. 99528 grs.
15. 91214 grs.
16. 137036 in.
17. 2181240 secs.
18. 17875 sq. in.
19. 643104 sq. in.
20. 99204 ft.
21. 101095 grs.
22. 1574624 drs.
23. 9591*f.*
24. 78256 grs.
25. 1358880 drs.
26. 506722*f.*
27. 673328 oz.
28. 58191 ft.
29. 85931 grs.
30. 25432235 secs.
31. 3055709 secs.

VIII.

1. £25 14*s.* 1*d.*
2. 428 tons 4 cwts. 13 lbs. 11 oz.
3. 90 lbs. 11 oz. 3 dwts. 17 grs.
4. 13 ac. 12 pls. 17 yds. 5 ft.
5. 48 wks. 3 days 13 hrs. 8 m. 55 secs.
6. 19 lbs. 4 oz. 3 dwts. 22 grs.
7. 5 mls. 3 fur. 38 pls. 2 yds. 8 in.
8. 204 tons 2 cwts. 2 qrs. 1 lb.

9. 1 wk. 10 hrs. 25 secs.
10. 25 loads.
11. 1041 gross 8 doz.
12. 7 ac. 26 pls. 10½ yds.
13. 4 lbs. 7 dwts. 13 grs.
14. 18 cwts. 8 lbs. 7 oz.
15. 91 mls. 3 fur. 15 pls. 5 yds. 1 ft. 3 in.
16. 3 mls. 7 fur. 9 pls. 2 yds. 1 ft. 5 in.
17. 46 lbs. 3 oz. 2 dwts. 2 grs.
18. 14 cub. yds. 2 ft. 1278 in.
19. 2 cub. yds. 26 ft. 57 in.
20. 49 mls. 2 fur. 31 pls. 3 yds. 1 ft. 11 in.
21. 2 tons 8 cwts. 1 qr. 18 lbs. 2 oz.
22. 21 cubic yds. 4 ft. 189 in.
23. 113067.
24. 9880 crowns.
25. 16048 half sovs. + 2*s.*
26. 11280 nls.
27. 43815 francs.
28. £4 17*s.* 7*d.*
29. 294.

IX.

1. £71516 14*s.* 7*d.*
2. £42610 15*s.* 8¼*d.*
3. 2152 cwts. 2 qrs. 16 lbs.
4. 297 cwts. 1 qr. 13 lbs.
5. 321 lbs. 4 oz. 19 dwts.
6. 374 lbs. 6 oz. 16 dwts.
7. 90 acres 23 pls.
8. 158 ac. 3 rds. 39 pls.
9. 279 ac. 1 rd. 7 pls.
10. 144 ac. 24 pls. 17 yds. 2¼ ft.
11. 656 deg. 38 m. 10 secs.
12. 3064 deg. 55 m. 36 secs.
13. 106 gals. 3 nags.
14. 131 gals. 2 qrts.
15. £153 13*s.* 5*d.*
16. £232 17*s.* 10*d.*
17. £218 19*s.* 9*d.*
18. £247 6*s.* 7¾*d.*
19. £234 8*s.* 8½*d.*
20. £1168 4*s.*
21. £176 19*s.* 6¾*d.*
22. £3867 8*s.* 11¾*d.*
23. £10011 12*s.* 0¾*d.*
24. £7436 16*s.* 0¼*d.*
25. £22898 9*s.* 2¼*d.*
26. £35856 4*s.* 10*d.*
27. £17041 5*s.* 3½*d.*
28. £10845 16*s.* 9¼*d.*
29. £12943 14*s.* 2*d.*
30. £1363 11*s.* 9½*d.*
31. 163 tons 9 cwts. 14 lbs.
32. 53 yds. 2 ft. 11½ in.
33. 165 hhds. 50 gals. 2 qrts.
34. 148 mls. 4 fur. 11 pls. 1 ft. 6 in.
35. 200 mls. 3 fur. 21 pls.
36. 138 hrs. 56 mins. 20 secs.
37. 340 yrs. 3 m. 2 wks. 4 days.
38. £190183 12*s.* 1*d.*
39. £120551 12*s.* 7*d.*
40. £261171 0*s.* 1¼*d.*
41. £205099 13*s.* 5½*d.*
42. £98606 5*s.* 8½*d.*
43. £219250 6*s.* 7½*d.*
44. £188558 4*s.* 1¼*d.*

115141 0*s.* 6½*d.*

231509 15*s.* 6*d.*

217749 12*s.* 1¼*d.*

46. £256478 0*s.* 4½*d.*

48. £109771 1*s.* 1¾*d.*

X.

89 8*s.* 7¾*d.*

966 16*s.* 9¾*d.*

87 13*s.* 8¼*d.*

1895 17*s.* 6¾*d.*

1012 2*s.* 9¼*d.*

79 cwts. 2 qrs. 25 lbs.

6 ac. 3 rds. 34 pls.

86 ac. 3 rds. 31 pls.

3 lbs. 14 oz. 15 drs.

81 lbs. 15 oz. 15 drs.

0 yrs. 11 m. 1 wk.

days 12 hrs. 52 min.

9 oz. 17 dwts. 21 grs.

lbs. 4 oz. 15 dwts.

yds. 2 ft. 11 in.

sq. yds. 7 ft. 114 in.

7 yds. 3 nls. 3 qrs.

53 6*s.* 6*d.*

13 4*s.* 9*d.*

2 8*s.* 6*d.*

966 2*s.* 2½*d.*

740 7*s.* 9¼*d.*

02 tons 18 cwts. 2 qrs. 20 lbs.

80898 10*s.* 9½*d.*

480259 12*s.* 6¾*d.*

310205 5*s.* 10¾*d.*

0 mls. 6 fur. 38 pls. 4 yds. 5 in.

2. £102 18*s.* 10¾*d.*

4. £1821 1*s.* 7¾*d.*

6. £103 16*s.* 0¾*d.*

8. £2693 0*s.* 8¾*d.*

10. 86 cwts. 3 qrs. 2 lbs.

12. 4 cwts. 2 qrs. 24 lbs.

14. 77 ac. 14 pls.

16. 110 ac. 2 rds. 15 pls.

18. 61 lbs. 14 oz. 12 drs.

20. 73 lbs. 2 oz. 13 drs.

22. 15 yrs. 10 m. 3 wks.

24. 42 days 21 hrs. 54 m.

26. 56 lbs. 10 oz. 14 dwts.

28. 1 oz. 16 dwts. 21 grs.

30. 6 mls. 7 fur. 34 pls.

32. 19 sq. pls. 25 yds. 2 ft. 16 in.

34. 47 ells 3 qrs. 2 nls. 2 in.

36. £44 13*s* 7½*d.*

38. £147 10*s.* 9¾*d.*

40. £247 11*s.* 7¾*d.*

42. £288 4*s.* 3¾*d.*

44. £577696 14*s.* 10½*d.*

46. 53 yds. 1 ft. 2 in.

48. £557862 12*s.* 6½*d.*

50. £319251 7*s.* 7½*d.*

52. 18 mls. 6 fur. 38 pls. 4 yds.

XI.

175 17*s.* 11*d.*

271 18*s.* 9*d.*

336 9*s.* 7*d.*

2. £201 7*s.* 9¼*d.*

4. £556 19*s.* 10½*d.*

6. £508 2*s.* 10½*d.*

7. £17039 11*s.* : £19169 9*s.* 10½*d.*
8. £28343 19*s.* : £29524 18*s.* 11½*d.*
9. £46079 5*s.* 4*d.* : £51839 3*s.* 6*d.*
10. £64695 11*s.* 3*d.* : £69008 12*s.*
11. £51345 17*s.* 6*d.* : £51773 15*s.* 1¾*d.*
12. £42830 19*s.* 6*d.* : £46724 14*s.*
13. £39225 19*s.* 11¼*d.* : £42068 9*s.* 2½*d.*
14. £32004 16*s.* 1¾*d.* : £33015 9*s.* 8½*d.*
15. £49454 5*s.* 9¼*d.* : £53786 1*s.* 0¼*d.*
16. £30324 4*s.* 1½*d.* : £48985 5*s.* 1½*d.*
17. 2930 acres 3 rds. 10 pls.
18. 12212 acres 8 pls.
19. 2188 cwts. 1 qr. 16 lbs.
20. 12021 cwts. 3 qrs. 27 lbs.
21. 14800 yds. 1 qr. 2 nls.
22. 27909 yds. 2 qrs. 1 nl.
23. £2502 14*s.* 7*d.*
24. 447 tons 3 cwts. 2 qrs. 15 lbs. 13 oz. 15 drs.
25. £54462 11*s.* 2*d.*
26. £362 15*s.* 1½*d.*
27. £4801 12*s.* 8*d.*
28. £37548 15*s.* 2*d.*
29. £531559 8*s.* 9*d.*
30. £13654 0*s.* 11*d.* : £17268 7*s.* 0½*d.*
31. £74963 10*s.* 2½*d.*
32. £377556 13*s.* 7*d.*
33. £609007 18*s.* 0½*d.*
34. £54259 15*s.* 9½*d.* : £63303 1*s.* 6¾*d.*
35. £1046104 19*s.* 0¾*d.*
36. £9744017 16*s.* 10½*d.*
37. £862287 5*s.* 7½*d.*

XII.

1. £13 19*s.* 2*d.*
2. £11 16*s.* 4½*d.*
3. £12 12*s.* 9½*d.*
4. £13 18*s.* 5¾*d.*
5. £11 11*s.* 10½*d.*
6. £12 18*s.* 6¾*d.*
7. £142 9*s.* 5¼*d.* : £116 6*s.* 0¾*d.* + 5¼*d.*
8. £112 0*s.* 8¼*d.* : £99 11*s.* 8½*d.* + 1*s.* 1½*d.*
9. £21 18*s.* 10½*d.* : £18 5*s.* 8¾*d.*
10. £10 17*s.* 9*d.* + 8¼*d.* : £17 5*s.* 6¾*d.* + 6¾*d.*
11. £34 18*s.* 3¼*d.* + 7¾*d.* : £29 7*s.* 10*d.*
12. £22 11*s.* 11¼*d.*
13. 75 cwts. 6 lbs. 7 oz. 7 drs. + 3 oz. 8 drs.
14. 43 cwts. 3 qrs. 17 lbs. 10 oz. 10 drs. + 8 oz. 4 drs.
15. 1 cwt. 2 qrs. 25 lbs. 2 oz. 3 drs. + 7 drs.
16. 3 acres 1 rd. 14 pls.
17. 7 acres 3 rds. 29 pls. 28 yds. 5 ft. + 2 ft. 72 in.
18. 495.

19. 1100.
20. 1465.
21. 1195.
22. 1175 + 7 lbs.
23. 965.
24. £71 15*s.* 11*d.* + 1*s.* 10½*d.*
25. £2 6*s.* 6½*d.* + 15*s.*
26. £17 11*s.* 10½*d.* + 10*s.* 6*d.*
27. £6 11*s.* 9½*d.* + £1 15*s.*
28. £1 3*s.* 8¼*d.* + 6*d.*
29. £0 4*s.* 8¾*d.* + 2¼*d.*
30. £10845 13*s.* 1*d.*
31. £981 6*s.* 4*d.*
32. £20270 4*s.* 11*d.*
33. £103899 0*s.* 3*d.*
34. £1625 12*s.* 9¼*d.* + 1*d.* : £1857 17*s.* 5½*d.* + ½*d.*
35. £366 0*s.* 9*d.*
36. £76 17*s.* 8¼*d.* + 1*s.* 6¼*d.*
37. £148805 3*s.* 1*d.*
38. £37200 11*s.* 3¼*d.*
39. £183 0*s.* 4½*d.*
40. £3246 9*s.* 7*d.*
41. £811 12*s.* 4¾*d.*
42. £597 4*s.* 0¾*d.*

XIII.

1. 46 square yards 5 feet 127½ inches.
2. 101 square yards 3 feet 90 inches.
3. 2 cubic feet 1457 inches.
4. 3 square yards 4 feet 96 inches.
5. 24 feet 10 inches.
6. 282 cubic feet 891·023624 inches.
7. 70 feet $3'\ 8''\ 10'''\ 4''''$.
8. $10124\frac{31}{36}$ sq. inches.
9. 9 ft. $2'\ 7''\ 2'''\ 5''''$.
10. 277 sq. yds. 7 ft. 11 in.

XIV.

1. £750 2*s.*
2. £15 6*s.* 5¾*d.*
3. 2*s.* 6*d.*
4. £986 0*s.* 9¾*d.*
5. £52 6*s.* 10½*d.* : £47 13*s.* 1½*d.*
6. £5 10*s.* 3*d.*
7. 1 qr. 25 lbs. 8 oz.
8. 3 yrs. 24 days.
9. £56 16*s.* 5¾*d.*
10. 13 ac. 3 rds. 24 pls.
11. 7 cwts. 3 qrs. 26 lbs.
12. £17 12*s.* 10¼*d.*
13. 146 dollars.
14. 5 tons 9 cwts. 3 qrs. 23 lbs. 7 oz. 6 drs. + 3 oz.
15. £110 5*s.*
16. £300.
17. 20 shirts.
18. 25 sts. 2 lbs. 8 oz.
19. 500 dollars.
20. 49 yds. 3 qrs. 2 nls.
21. £8 13*s.* 4*d.*
22. £175 15*s.* 8¼*d.* : £125 3*s.* 2¼*d.*
23. £5372 4*s.* 7¾*d.*
24. 313632000 sq. inches.

25. 570240 inches.
26. 41847 farthings.
27. 662832 oz.
28. £25 5*s*. 9$\frac{3}{8}$*d*.
29. £58 15*s*.
30. £257 0*s*. 4$\frac{1}{4}$*d*.
31. 224461$\frac{1}{2}$ feet.
32. 18 lbs. 2 oz. 18 dwts. 3 grs.
33. 93 tuns 2 hhds. 17 gals. 3 qrts. 1$\frac{2}{5}$ pt.
34. £5506 5*s*. : £4958 15*s*.
35. 2 miles an hour.
36. £1 7*s*.
37. 273 ft. 7′ 5″ 3‴.
38. One-twelfth of a cubic foot.
39. 107224$\frac{8}{13}$ times.
40. 3 cub. yds. 20 ft. 1455 in.
41. 179685 sq. feet
42. 2 mls. 5 fur. 39 pls. 1 yd. 4 in.
43. 39 lbs. 8 ozs. 17 dwts. 20 grs.
44. 278172 inches.
45. 9 tons 2 cwts. 2 qrs. 26 lbs. 15 oz.
46. 4221 feet.
47. 16 hrs. 20 mins. 18 secs.
48. 514752 ozs.
49. 59 mls. 5 furs. 34 pls. 1 ft. 9 in.
50. 229168 ozs.
51. $\frac{1}{2}$ hour.
52. 27 mls. 1 fur. 15 pls. 5 yds. 3 in.
53. 3 yds. 1 ft.
54. 35101$\frac{1}{2}$ feet.
55. 592 sq. yds. 4 ft. 106 in.
56. 192669 feet.
57. 1820752 drams.
58. 85931 grs.
59. 27 cwts. 1 qr. 23 lbs. 7 ozs.
60. 615187 cubic inches.
61. 49602 feet.
62. 37$\frac{1}{2}$ lbs.
63. 29 mls. 1 fur. 11 pls. 2 yds. 2 ft. 4 in.
64. 17526.
65. 266450 grs.
66. 632592 ozs.
67. 8 mls. 3 fur. 32pls. 3 yds. 2 in.
68. 1028 tons 1 cwt. 3 lbs. 11 oz.
69. 115 tons 17 cwts. 2 qrs. 6 lbs.
70. 2 wks. 6 days 5 hrs. 52 m. 14 secs.

XV.

1. $7\frac{2}{3} : 4\frac{1}{7} : 1\frac{16}{17}$.
2. $54\frac{12}{25} : 212\frac{29}{33} : 72\frac{9}{11}$.
3. $14\frac{29}{144} : 103\frac{12}{19} : 111\frac{16}{21}$.
4. $9\frac{1}{2} : 21\frac{8}{17} : 13\frac{38}{47}$.
5. $\frac{64}{9} : \frac{38}{5} : \frac{129}{7}$.
6. $\frac{5171}{40} : \frac{387}{23} : \frac{52}{9}$.
7. $\frac{316}{17} : \frac{351}{5} : \frac{764}{11}$.
8. $\frac{289}{9} : \frac{199}{12} : \frac{13564}{119}$.
9. $\frac{209}{11} : \frac{170}{17} : \frac{870}{29}$.
10. $\frac{1053}{39} : \frac{1794}{46} : \frac{9405}{57}$.
11. 22 : 39.
12. 9 : 4.

13. 5 : 4.
14. 6 : 19.
15. 36 : 8.
16. 14 : 8.
17. $\frac{3}{7} : \frac{3}{10} : \frac{2}{11}$.
18. $\frac{7}{103} : \frac{4}{5} : \frac{9}{11}$.
19. $\frac{9}{13} : \frac{4}{9} : \frac{5}{7}$.
20. $\frac{1}{6} : \frac{5}{9} : \frac{11}{13}$.
21. $\frac{20}{21}$.
22. $\frac{7}{8}$.
23. $\frac{5355}{7144}$.
24. $\frac{12}{24}, \frac{16}{24}, \frac{20}{24}, \frac{21}{24}$.
25. $\frac{36}{48}, \frac{28}{48}, \frac{15}{48}, \frac{42}{48}$.
26. $\frac{90}{330}, \frac{255}{330}, \frac{264}{330}, \frac{190}{330}$
27. $\frac{6545}{8415}, \frac{6120}{8415}, \frac{7983}{8415}, \frac{4465}{8415}$.
28. $\frac{360}{630}, \frac{350}{630}, \frac{385}{630}, \frac{546}{630}$.
29. $\frac{7}{8} : \frac{63}{80} : \frac{3}{32}$.
30. $\frac{11}{80} : \frac{13}{160} : \frac{37}{80}$.
31. $\frac{3}{4} : \frac{5}{8} : \frac{3}{8}$.
32. $\frac{5}{8} : \frac{7}{8} : \frac{5}{16}$.
33. $\frac{5}{8} : \frac{13}{16} : \frac{7}{16} : \frac{29}{32}$.
34. $\frac{3}{7}$.
35. $\frac{7}{11}$.
36. $\frac{11}{13}$.

XVI.

1. $1\frac{1}{4} : 2\frac{4}{7}$.
2. $2 : 2\frac{3}{13}$.
3. $1\frac{17}{60}$.
4. $1\frac{73}{84}$.
5. $1\frac{29}{36}$.
6. $1\frac{17}{28}$.
7. $1\frac{2341}{3120}$.
8. $1\frac{35}{36}$.
9. $2\frac{31}{36}$.
10. $1\frac{8}{21}$.
11. $2\frac{37}{660}$.
12. $4\frac{14}{15}$.
13. $3\frac{65}{72}$.
14. $5\frac{71}{168}$.
15. $7\frac{8}{9}$.
16. $2\frac{151}{385}$.
17. $10\frac{22}{45}$.
18. $8\frac{11}{18}$.
19. $13\frac{61}{120}$.
20. $5\frac{89}{120}$.
21. $5\frac{31}{40}$.
22. $6\frac{7}{12}$.
23. $14\frac{25}{84}$.

XVII.

1. $\frac{1}{4} : \frac{1}{3} : \frac{2}{7}$.
2. $\frac{1}{7} : \frac{3}{19} : \frac{3}{8}$.
3. $\frac{47}{110}$.
4. $\frac{1}{40}$.
5. $2\frac{5}{21}$.
6. $4\frac{67}{90}$.
7. $\frac{41}{100}$.
8. $2\frac{1}{5}$.
9. $2\frac{22}{35}$.
10. $2\frac{81}{140}$.
11. $10\frac{5}{21}$.
12. $3\frac{1}{3}$.
13. $12\frac{7}{8}$.
14. $3\frac{35}{36}$.
15. $3\frac{179}{180}$.
16. $2\frac{19}{60}$.
17. $21\frac{1}{20}$.

XVIII.

1. $\frac{12}{55}$.
2. $\frac{1}{6}$.
3. $\frac{20}{63}$.
4. $16\frac{31}{84}$.
5. $\frac{15}{224}$.
6. $5\frac{29}{32}$.
7. $\frac{34}{315}$.
8. $97\frac{3}{5}$.
9. $443\frac{3}{32}$.
10. 84.
11. $99\frac{1}{12}$.
12. $253\frac{15}{28}$.
13. 30.
14. $40\frac{71}{84}$.
15. $96\frac{3}{35}$.
16. 91.
17. $15\frac{7}{9}$.
18. 56.
19. 2*s.* 11*d.*
20. £3 12*s.* 2*d.*
21. 8*s.* 7*d.*
22. 63.

XIX.

1. $\frac{16}{63} : \frac{3}{4}$.
2. $\frac{20}{51} : 13\frac{3}{4}$.
3. $1\frac{31}{36} : 1\frac{79}{92}$.
4. £8 6*s.* $4\frac{4}{11}$*d.*
5. £21 8*s.* $6\frac{6}{7}$*d.*
6. 23 cwts. 1 qr. $2\frac{4}{5}$ lbs.
7. 5 ac. 1 rd. 17 pls. 15 yds. 1 ft. 18 in.
8. $1\frac{2}{3} : \frac{21}{80} : 13\frac{1}{2}$.
9. $\frac{5}{32} : 1\frac{3}{13} : 32\frac{2}{3}$.
10. $\frac{7}{20}$.
11. $\frac{9}{10}$.
12. $\frac{1}{98}$.
13. $3\frac{6}{7}$.
14. $1\frac{3}{5}$.
15. $\frac{15}{16}$.
16. $\frac{1}{2}$.
17. $\frac{41}{189}$.
18. $\frac{1}{8}$.
19. $2\frac{11}{272}$.
20. $\frac{169}{306}$.
21. $1\frac{295}{351}$.
22. $\frac{34}{65}$.
23. $\frac{87}{115}$.
24. $1\frac{38}{43}$.
25. $3\frac{5}{9}$.
26. $1\frac{9}{22}$.
27. $\frac{16}{27}$.
28. $\frac{13}{25}$.
29. $10\frac{1}{25}$.
30. $\frac{59}{120}$.
31. 15*s.* 10*d.*
32. $5\frac{1}{3}$ inches.

XX.

1. ·095.
2. ·039.
3. 12·068.
4. ·609.
5. 50·06.
6. 29·009.
7. 47·4.
8. 69·607064.
9. 120·014019.
10. 55·6494.
11. 60·06006.
12. 17·017017.

XXI.

1. 425·9573.
2. 757·499.
3. 47·0709.
4. 19·6338.
5. 6404·8502.
6. 448·8074.
7. 183·0475.
8. 564·282692.
9. 779·1686.
10. 580·491902.
11. 7999·77021.
12. 515·92373.
13. 12988·7753.

XXII.

1. 118·114.
2. 975305·1258.
3. 109997342·809099.
4. 11·7537.
5. 12·561.
6. 446·352.
7. 115·2839.
8. 2·109.
9. 3011·5026.
10. 1126·849608.
11. 29738·94371.
12. 1637·269485.
13. 33·06886.

XXIII.

1. 510·3.
2. 375·54564.
3. 146·4561.
4. 341·508486.
5. ·0037777.
6. 1·69194
7. ·37368892.
8. ·2460168.
9. 2·691066.
10. 11961·232.
11. 636·83564.
12. ·035594.
13. ·7625 : ·07392.
14. ·2760494.
15. 5424·8324832.
16. ·01977048.
17. 32·5819809.
18. ·3154032.

XXIV.

1. 12·96.
2. 7·8125.
3. 185000.
4. 12·932...
5. ·021...
6. 7 tons 18 cwts. 3 qrs. 10 lbs. 8 oz. 7·168 drs.
7. £160 14*s.* 5½*d.*
8. 24106·25.
9. 3·271...
10. ·15.
11. 66·993.
12. 686·769...

13.* $13333\cdot\dot{3}$.
14. 74·782...
15. $12054\cdot\dot{0}5\dot{4}$.
16. 4000.
17. $399016\cdot\dot{1}4\dot{8}$.
18. ·6586...
19. 50.
20. $32\cdot9\dot{4}1552051\dot{1}$
21. ·013 : ·0013 : 130.
22. ·01026...
23. 1230·769...
24. $\frac{1}{80}$.
25. $\frac{7}{4000}$.
26. $2\frac{89}{2000} : \frac{1}{1600}$.
27. $23\frac{61}{1600} : \frac{11}{16000}$.
28. ·5 : ·2 : ·875.
29. ·4375 : ·44 : ·8125.
30. ·790625 : ·8875.
31. ·40625£.
32. $\cdot\dot{3}$ of a ton.
33. $\cdot148\dot{8}0952\dot{3}$ of 21*s.*
34. $\cdot2\dot{9}3650\dot{7}$ of a guinea.
35. ·615625 of a ton.
36. ·09375.
37. $1\cdot\dot{2}\dot{1}$.
38. ·078125 : $\cdot2\dot{1}4285\dot{7}$.
39. ·084.
40. ·875£ : ·68125£.
41. ·840625£ : ·975£.
42. ·890625£ : ·396875£.
43. ·54375£ : ·109375£.
44. 15*s.* 4·536*d.* : 5·58*d.*
45. 23 hrs. 25 mins. 26·4 secs.
46. 4*s.* 2·72*d.*
47. 3 rds. 7 pls. 9 yds. 8 ft. 42·912 in.
48. 13*s.* $1\frac{3}{4}$*d.* : 19*s.* $6\frac{3}{4}$*d.*
49. £5 10*s.* 6*d.*
50. £1 6*s.* 11·856*d.*
51. 15*s.*
52. 5*s.*
53. 14*s.* $7\frac{1}{2}$*d.*
54. 6*d.*
55. $5\frac{1}{4}$*d.*

XXV.

1. $\frac{7}{9} : \frac{2}{3}$.
2. $\frac{5}{11} : \frac{5}{37}$.
3. $\frac{5}{18} : \frac{13}{22}$.
4. $\frac{7}{12} : \frac{9}{13}$.
5. $\frac{11}{15} : \frac{5}{6}$.
6. $\frac{2}{11} : \frac{19}{24}$.
7. $\cdot1009\dot{5}6\dot{9}$.

XXVI.

1. £13786 7*s.* 6·24*d.*
2. 3537 tons 17 cwts. 13 lbs. 7·04 oz.
3. £255 6*s.* 0·96*d.*
4. 5927 cwts. 14·56 lbs.
5. $\frac{7}{12}$.
6. 6.
7. ·0101...
8. $2\frac{23}{48}$.
9. 14·49.
10. $1\cdot0\dot{4}2857\dot{1}$.
11. 2 9303.
12. $4\frac{20}{49}$.

---

* For the explanation of the dots placed over the figures, see CIRCULATING DECIMALS.

13. £2 12*s.* 6*d.*
14. $\frac{117}{119}$.
15. $\frac{184}{915}$.
16. $9\frac{4}{5}$*d.*
17. $\frac{1785}{3572}$.
18. $\frac{26}{35}$.
19. ·034102562.
20. $\frac{677}{1731}$.
21. 3.
22. 17*s.* 8*d.*
23. $\cdot\dot{0}7692\dot{3}$.
24. $\frac{36}{275}$.
25. 13400 : ·00134.
26. ·09375.
27. 3 qrs. 10 lbs. 14·3616 oz.
28. 1433·44017.
29. $2\frac{23}{30}$.
30. 10*s.* 6*d.*
31. 39.
32. £4 2*s.* $1\frac{33}{35}$*d.*
33. 344·32994103.
34. ·0190752.
35. ·015625.
36. $\cdot1041\dot{6}$.
37. $\frac{403}{2220}$.
38. $\cdot38541\dot{6}$.
39. 8·14968*d.*
40. 45.
41. $1\frac{26}{87}$.
42. $10556\cdot4673\dot{7}1428\dot{5}$:
$105\cdot564673\dot{7}1428\dot{5}$.
43. ·39375£.
44. $\cdot\dot{0}7\dot{4}$.
45. 93·00365.
46. 27·00235.
47. 1980·13800195.
48. 1·81823...

XXVII.

1. £2 12*s.* $10\frac{1}{2}$*d.*
2. £2 4*s.*
3. £33 8*s.* 3*d.*
4. £93 2*s.* $0\frac{3}{4}$*d.*
5. £36 14*s.* 3*d.*
6. £5378 19*s.* $10\frac{1}{2}$*d.*
7. £1544 3*s.* $10\frac{1}{4}$*d.*
8. £738 13*s.*
9. £17289 0*s.* $6\frac{3}{8}$*d.*
10. £9643 6*s.* $0\frac{3}{4}$*d.*
11. £1912 14*s.*
12. £179 17*s.* $10\frac{3}{4}$*d.*
13. £258 17*s.* 1*d.*
14. £367 3*s.* $10\frac{1}{2}$*d.*
15. £1060 6*s.* $7\frac{1}{4}$*d.*
16. £625 19*s.* $0\frac{3}{8}$*d.*
17. £184 14*s.* 8*d.*
18. £67 8*s.* 6*d.*
19. £738 13*s.*
20. £34596 9*s.* $8\frac{1}{4}$*d.*
21. £91949 1*s.* $10\frac{11}{16}$*d.*
22. £78 6*s.*
23. £34 9*s.* $9\frac{1}{2}$*d.*
24. £1030 3*s.* 9*d.*
25. £31907 9*s.* 11*d.*
26. £10757 19*s.* 9*d.*
27. £669 2*s.* $9\frac{3}{4}$*d.*
28. £49 14*s.* $1\frac{1}{2}$*d.*
29. £241 10*s.*
30. £100 15*s.* $7\frac{1}{2}$*d.*
31. £59 19*s.* $9\frac{3}{56}$*d.*
32. £311 8*s.*
33. £87 0*s.* $7\frac{1}{2}$*d.*
34. £83 6*s.* $5\frac{1}{4}$*d.*
35. 10*s.* $1\frac{17}{80}$*d.*
36. £909 9*s.* 3*d.*
37. £25557 18*s.* $1\frac{1}{2}$*d.*
38. £641 14*s.* $4\frac{1}{2}$*d.*

39. £3282 2s. 8¼d.
40. £99 18s.
41. £1381 15s. 0¾d.
42. £664 0s. 7½d.
43. £1073 11s. 11¾d.
44. £50 2s. 3d.
45. 18 cwts. $4\frac{11}{50}$ lbs.
46. 128 cwts. 1 qr. $11\frac{2}{35}$ lbs.
47. £12 15s.
48. 140 yds.
49. £4502 1s. 9¼d.
50. £2 5s.
51. 18s.

XXVIII.

1. £223 11s. 3d.
2. £1912 14s.
3. £10299 1s. 8d.
4. £138 14s. 4d.
5. £1204 17s. 6d.
6. £2976 16s. 8d.
7. £1828 18s. 8d.
8. £1727 4s. 6¾d.
9. £245 5s. 5d.
10. £37 16s. 3d.
11. £216 19s. $5\frac{7}{16}$d.
12. £417 3s. 9d.
13. £55 19s. 2¾d.
14. £1581 9s. 0¼d.
15. £63 6s. 5¾d.
16. £158 16s. 6d.
17. £190 9s. 1½d.
18. £966 19s. 7½d.
19. £411 17s. 10d.
20. £20618 11s. 2d.
21. £91942 17s. $6\frac{9}{16}$d.
22. £9571 15s. $0\frac{9}{56}$d.
23. £377 0s. $6\frac{3}{28}$d.
24. £1375 13s. 4¾d.
25. £82 0s. 7½d.
26. £327 17s. $6\frac{15}{16}$d.
27. £9 6s. 8d.
28. £7 4s. $4\frac{13}{16}$d.
29. £360 12s. 6d.
30. £419 3s. 4½d.*
31. £2238 16s. 9¼d.
32. £25 5s. 9⅝d.
33. £10 18s. 2¾d.
34. £51 1s. 8½d.
35. £241 10s.
36. £2210 12s. 3½d.
37. £431 13s. 7½d.
38. £269 17s. 8·9d.
39. £910 14s. 3½d.
40. £93 14s. 4¾d.
41. £370703 2s. 6d.
42. £2025 9s. 11½d.
43. £1073 11s. 11¾d.
44. £11586 2s. 7½d.
45. £24 15s. $1\frac{27}{32}$d.
46. £2210 12s. 3½d.
47. £227 14s.
48. £897 0s. 11½d.
49. £4724 6s. 1½d.
50. £343 12s. 4¾d.
51. £570 8s. 5¾d.
52. £36 10s. 2¼d.
53. £154 10s. 9d.

XXIX.

1. 450 acres.
2. 1 wk. 1 day 9 hrs. 36 mins.
3. £133 6s. 8d.
4. 400 men.

---

* A month is here taken as 30 days.

5. 45 days.
6. $13\frac{1}{2}$ weeks.
7. 10 hrs.
8. £15 18*s*. $11\frac{52831}{131867}$*d*.
9. £384 15*s*. 5*d*.
10. £7 7*s*. $6\frac{66}{73}$*d*.
11. 5*s*. $1\frac{167}{244}$*d*.
12. $3\frac{3}{4}$ hrs.
13. £179 7*s*. $9\frac{1}{2}$*d*.
14. £496 14*s*. $3\frac{3}{4}$*d*.
15. £402 0*s*. $11\frac{1}{2}$*d*.
16. £5583 6*s*. 8*d*.
17. £2380 10*s*.
18. £30 14*s*. 3*d*.
19. £100 9*s*. $4\frac{164}{449}$*d*.
20. £211 19*s*. 3*d*.
21. £25 17*s*. $1\frac{11625}{11651}$*d*.
22. £12 11*s*. $10\frac{66}{425}$*d*.
23. 21 cwts. 3 qrs. 21 lbs.
24. £57 6*s*. $8\frac{147}{294}$*d*.
25. £12 2*s*. 8*d*.
26. 35 cows.
27. 77 cwts. 1 qr. $22\frac{10}{11}$ lbs.
28. £66 10*s*.
29. 460 ac. 3 rds. 24 pls.
30. $82\frac{1}{2}$ yds.
31. £189 14*s*. 11*d*.
32. £83 11*s*. 3*d*.
33. 5 cwts. 2 qrs. 14 lbs.
34. £31 6*s*. $9\frac{3}{7}$*d*.
35. £3 10*s*.
36. £75.
37. £90.
38. $10{\cdot}431\dot{8}$*d*.
39. 1*s*. 8*d*.
40. £1 8*s*. 9*d*. : £1 3*s*. : 5*s*. 9*d*.
41. £19 9*s*. 2*d*. : £25 18*s*. $10\frac{2}{3}$*d*. : £32 8*s*. $7\frac{1}{3}$*d*.
42. £144 and £128.
43. £11 0*s*. 10*d*. : £8 16*s*. 8*d*.

XXX.

1. £2 5*s*. $8\frac{1}{3}$*d*.
2. 126 acres 1 rd. $5\frac{5}{7}$ pls.
3. £5.
4. £239 16*s*. $4\frac{4}{5}$*d*.
5. 16 days.
6. 2340 lbs.
7. £100.
8. 10 hrs.
9. 6440 men.
10. $13\frac{1}{2}$ days.
11. 8 hrs.
12. $36\frac{2}{3}$ lbs.

XXXI.

1. £60 12*s*. $3\frac{3}{16}$*d*.
2. £35 0*s*. $9\frac{204}{349}$*d*.
3. £8 18*s*. $6\frac{3}{4}$*d*.
4. £25 17*s*. $1\frac{11625}{11651}$*d*.
5. 1 year 2 months.
6. £12 16*s*. 4*d*.
7. 3 oz.
8. 8064 bricks.
9. £123 10*s*. $2\frac{6}{7}$*d*.
10. £7237 6*s*. 8*d*.
11. £299 5*s*.
12. £8 14*s*. $0\frac{9}{16}$*d*.
13. £71 14*s*. $4\frac{7}{8}$*d*.
14. 3 lbs. $4\frac{1}{2}$ oz.
15. 6*s*.
16. £767.
17. £90.
18. $46\frac{29}{40}$ sovereigns.
19. £2257 10*s*.
20. 5*s*.

21. £3.
22. £18 18*s.*
23. £14 3*s.* 4*d.*
24. £8 15*s.*
25. 2894 francs 3 cents.
26. 11 cwts. 1 qr.
27. £1736 4*s.* $7\frac{1}{2}$*d.*
28. £502 0*s.* $11\frac{1}{2}$*d.*
29. $7\frac{1}{2}$ days.
30. 64 yards.
31. £2 10*s.* $10\frac{1}{2}$*d.*
32. £2881 16*s.* 8*d.*
33. £3848 12*s.* $11\frac{4}{7}$*d.*
34. £2143 8*s.* $10\frac{1}{2}$*d.*
35. £25 5*s.* $9\frac{3}{8}$*d.*
36. 6440 men.
37. $73\frac{1}{44}$.
38. 12·527.
39. 18·7 lbs.
40. $5\frac{1}{4}$*d.*
41. £242 9*s.* $4\frac{3}{4}$*d.*
42. £3.
43. 16 days.
44. 2 cwts. 3 qrs. 14 lbs.
45. 4 oz. $13\frac{1}{2}$ drs.
46. £75 5*s.* $5\frac{47}{243}$*d.*
47. 2*s.* $2\frac{4617}{9056}$*d.*
48. £7.
49. $58\frac{5}{22}$.
50. £27 8*s.* $6\frac{3}{4}$*d.*
51. £2417.
52. 9044 lbs. 6 oz. 7 dwts. $4\frac{4}{13}$ grs.
53. £42 9*s.* $4\frac{23}{28}$*d.*
54. £174 12*s.*
55. £55 3*s.* $6\frac{1}{4}$*d.*
56. £27 0*s.* $2\frac{2367}{4312}$*d.*
57. £28 2*s.* $6\frac{3}{4}$*d.*
58. £1849 11*s.* 6*d.*
59. $38\frac{2}{3}$ hrs.*

XXXII.

1. 30 gals.
2. 7 doz.
3. $5\frac{1}{3}$ lbs.

XXXIII.

1. £600.
2. A, £60 : B, £72 : C, £78.
3. A, £13 4*s.*, B, £19 16*s.*, C, £26 8*s.*, D, £21 9*s.*

XXXIV.

1. £65 19*s.* 6*d.*
2. £289 5*s.* $7\frac{4}{5}$*d.*
3. £76 17*s.* $9\frac{1}{3}$*d.*
4. £36 18*s.* 1*d.*
5. £687 2*s.* $10\frac{1}{5}$*d.*
6. £304 13*s.* 9*d.*
7. £307 7*s.* $3\frac{3}{80}$*d.*
8. £7 14*s.* $1\frac{47}{73}$*d.*
9. £38 14*s.* $11\frac{17}{20}$*d.*
10. £438 8*s.* 0·165*d.*

---

* This question contains no misprint, as some suppose, for I had a copy of it verbatim before the Eighth Civil Service Report appeared.

11. £17 11*s*. $4\frac{41}{125}$*d*.
12. 3 per cent.
13. $4\frac{1}{4}$ per cent.
14. 4 per cent.
15. $4\frac{3}{4}$ per cent.
16. 8 years.
17. 4 yrs. 6 months.
18. £42 5*s*. 10*d*.
19. £1411 9*s*. 2*d*.
20. £277 7*s*. 6·3*d*.
21. £173 2*s*. 9*d*.
22. £1604 5*s*.
23. £34 17*s*. $5\frac{1}{2}$*d*.
24. $2\frac{1}{4}$ per cent.
25. £214 7*s*. $1\frac{5}{11}$*d*.
26. $2\frac{14849}{15640}$.
27. £95 12*s*. 1*d*.
28. 3 years.
29. £195 1*s*. $10\frac{1}{2}$*d*.
30. $3\frac{3}{5}$ per cent.

XXXV.

1. £13 10*s*. : £730.
2. 13*s*. $5\frac{119}{821}$*d*.
3. 16*s*. 8*d*.
4. £500.*
5. £2350.
6. £43 15*s*. 11*d*.
7. £21 19*s*. $1\frac{52}{89}$*d*.
8. £173 17*s*. $5\frac{48}{113}$*d*.
9. £260 16*s*. $2\frac{31}{226}$*d*.
10. £424 6*s*. $10\frac{2}{19}$*d*.
11. £300.

XXXVI.

1. £390.
2. £29 14*s*. $8\frac{23}{50}$*d*.
3. £57 11*s*. $7\frac{1}{8}$*d*.
4. £5 12*s*. 6*d*.
5. £4 17*s*. $8\frac{1}{4}$*d*.
6. £20 10*s*. 6*d*.
7. £57 18*s*. $3\frac{3}{5}$*d*.
8. £4736 16*s*. $10\frac{2}{19}$*d*.
9. £1900.
10. £52 7*s*. 9*d*.
11. £76 3*s*. 2·65*d*.

XXXVII.

1. £581 3*s*. $9\frac{5}{7}$*d*.
2. $7\frac{7}{18}$ months.
3. $8\frac{2}{7}$ months.
4. $4\frac{1}{3}$ months.
5. $12\frac{1}{5}$ months.

XXXVIII.

1. £8925.
2. £7200.
3. £6400.
4. 19 shares.
5. 160 shares.
6. £1627 7*s*. $0\frac{3}{4}$*d*.
7. £1111 10*s*.
8. £1187 3*s*. $1\frac{1}{2}$*d*.
9. £16869 9*s*. $7\frac{1}{2}$*d*.
10. £5173 5*s*. 9·96*d*.

---

* The explanation in such answers, for obvious reasons, is omitted.

11. £107 5*s.*
12. £96 0*s.* 5$\frac{55}{589}$*d.*
13. £160 6*s.* 8*d.*
14. £201 19*s.* 6$\frac{418}{443}$*d.*
15. £98 2*s.* 6*d.*
16. 3$\frac{4}{7}$: £4 8*s.* 10$\frac{2}{3}$*d.*
17. £3 19*s.* 6$\frac{6}{11}$*d.* : £4 3*s.* 4*d.*
18. £4 14*s.* 8$\frac{16}{169}$*d.*
19. £4.
20. £600 16*s.* 8$\frac{200}{719}$*d.*
21. Increase of £8 8*s.* 9$\frac{3}{5}$*d.*
22. £587 15*s.* 1$\frac{11}{49}$*d.*
23. £75.
24. 60 per cent.
25. Decreased £34 7*s.* 6*d.*
26. Increase of £5 18*s.* 4*d.*
27. Increased £16 6*s.* 2$\frac{82}{187}$*d.*
28. £24960.
29. £26 13*s.* 4*d.*
30. Increase of £115 4*s.*
31. £124 4*s.* 3$\frac{13}{17}$*d.*
32. Interest of 1st £3 7*s.* 1$\frac{85}{143}$*d.* per cent.; interest of 2nd £4 17*s.* 5$\frac{251}{821}$*d.* per cent.

XXXIX.

1. £4 5*s.* 10$\frac{2}{25}$*d.* per cwt.
2. 6*s.* 6$\frac{6}{23}$*d.*
3. 15$\frac{1}{2}$ per cent.
4. £156 12*s.*
5. £75 : loss £24.
6. 19*s.* 8*d.*
7. £65 9*s.* 11*d.*
8. £476 6*s.* 8*d.*
9. 272 cwts. 2 qrs. 14 lbs.
10. 4*s.* 8*d.* and 5*s.* 3*d.*
11. 10*s.* 6*d.* and £8 5*s.* 4$\frac{32}{127}$*d.*
12. £18 15*s.* per cent. gain.
13. 1*s.* 1$\frac{4}{9}$*d.*
14. £14 11*s.* 8*d.*

XL.

1. 19*s.* 2*d.*
2. 44.
3. 1541.
4. £58 1*s.* 4*d.*
5. 25313·48.
6. 63$\frac{7}{11}$.
7. $40\cdot\dot{6}$.
8. 12 per cent.
9. 18*s.* 9*d.*
10. £250 5*s.* 9$\frac{3}{4}$*d.*
11. 10$\frac{1}{12}$*d.*
12. 120000.
13. 3·129896477...
14. £2107 10*s.*
15. 1420·479... : 4326·5186... : 5705·042...
16. 12·403 : 37·779... : 49·816...

XLI.

1. 10648 : 900.
2. ·00000256 : 12·25.
3. 429981696 : 1·2762815625.
4. 85184 : $\frac{8}{27}$.
5. 46656 : 8000.
6. 134689 : 130321.

XLII.

1. 53·43.
2. 637·4.
3. $16\frac{1}{5}$.
4. $2\frac{4}{15}$ ($2\cdot2\dot{6}$).
5. $5\frac{4}{5}$.
6. $22\frac{11}{16}$.
7. 7047.
8. 2·6463.
9. 1137·4023.

XLIII.

1. 7093.
2. 97·02
3. 8767.
4. $17\frac{2}{7}$.
5. 2·924.
6. 512.
7. 23.
8. 611.
9. 79.
10. 1061.

XLIV.

1. 165.
2. 6.
3. 54.
4. $\frac{3}{4}$.
5. $\frac{4}{5}$.
6. 172·5.
7. 99.
8. $\cdot\dot{2}8571\dot{4}$.
9. $193\frac{5}{16}$.
10. $46\frac{7}{8}$.
11. 5460.
12. 1024.
13. $40\cdot\dot{4}8\dot{1}$.
14. 1.

XLV.

1. £765.
2. £328 17*s.* 7*d.*
3. £150.
4. £220 4*s.* 1*d.* nearly.
5. £500.
6. £5264 4*s.* $2\frac{5}{32}$*d.*
7. £334 5*s.* $3\frac{5}{16}$*d.*
8. £270 2*s.* 5·307456*d.*
9. £668 10*s.* 6·825*d.*
10. £10653 17*s.* 10·07889*d.*
11. £254 18*s.* 5·75784375*d.*
12. £9091 16*s.* 1·782075*d.*

XLVI.

1. Its integral part.
2. 1·3802113 : 1·8750613 and $\bar{1}\cdot6930606$.

3. ·6989700 : 1·1760913 : $\bar{1}$·1760913.
4. 14 digits ($512 = 2^9$): ·256.
5. For *Ans.* see **168**.
6. 10, see **174**.
7. 1·800786 : $\bar{2}$·199214.
8. ·7269987.

XLVII.

1. 8606.
2. 10942.
3. 2121.
4. 2988.
5. ·705.
6. 7371.
7. 34.
8. $21\frac{10}{23}$.

XLVIII.

1. 12049.
2. 23059.
3. 14522.
4. 13258.
5. 27258.
6. 57899.

XLIX.

1. 13744.
2. 11190.
3. 14482.
4. 16501.

L.

1. 70103.
2. Six hundred and seven thousand and thirty-five.
3. 340703529.
4. £22689 17*s.* $8\frac{1}{2}$*d.*
5. 40018178.
6. £1360 4*s.* $7\frac{3}{4}$*d.*
7. 1457558704.
8. £75834 19*s.* 8*d.*
9. £710150 12*s.* $9\frac{3}{4}$*d.*
10. $367478\frac{1}{7}$.
11. $49095\frac{3}{7}$.
12. £489 7*s.* 11*d.*

LI.

1. 533994.
2. 201995.
3. 105543798.
4. £25052 9*s.* 6*d.*
5. 19487107.
6. £2731 17*s.* 7*d.*
7. 167 tons 6 cwts. 1 qr. 23 lbs.
8. 44 tons 11 cwts. 1 qr. 23 lbs.

9. 85 lbs. 10 oz. 18 dwts. 21 grs.
10. 397712002623.
11. £33798 4*s.* $9\frac{1}{4}$*d.*
12. 201 tons 17 cwts. 3 qrs. 16 lbs.
13. £2800277 15*s.* 10*d.*
14. $150695441175\frac{1}{5}$.
15. $1992596033\frac{137}{341}$.
16. 1 ft. $5\frac{11}{12}$ inches.
17. £26389 4*s.* $9\frac{1}{4}$*d.* + 1*s.* $6\frac{3}{4}$*d.*
18. 111510 lbs.
19. 943197 pints.
20. 25 loads.
21. 40704 quarts.

LII.

1. 400000510.
2. Seventy-seven million five thousand and three.
3. 194197091.
4. £12817 11*s.* 4*d.*
5. 1153916042.
6. £524139 16*s.* $8\frac{3}{4}$*d.*
7. 48701164178112.
8. £218460 6*s.* $2\frac{1}{4}$*d.*
9. £366786 0*s.* 2*d.*
10. $1140599139\frac{290}{467}$.
11. £60653 16*s.* $1\frac{1}{2}$*d.*
12. £412 0*s.* $9\frac{3}{4}$*d.*
13. 23061 oz.
14. 137 miles 3 fur. 26 pls. 3 yds. 1 ft. 6 in.
15. £4595 8*s.* 4*d.*
16. $8\frac{7}{8}$ guineas (£9 6*s.* $4\frac{1}{2}$*d.*)
17. $16\frac{5}{16}$.
18. $11\frac{47}{60}$.
19. 81.
20. 4.
21. 1404·0287.
22. 406·1891.
23. 158466·011.
24. 1·137...
25. $548571\cdot\dot{4}2857\dot{1}$.
26. ·46875£.
27. 2·924.
28. 37 ft. $2' \; 2'' \; 8''' \; 1'''' \; 4'''''$
29. 37 cubic ft. $320\frac{1}{9}$ inches.
30. £9 16*s.* 8*d.*
31. 3*s.* 4*d.*
32. £41606 12*s.*
33. £40764 10*s.* 3*d.*
34. £59416 11*s.* 4*d.*

LIII.

1. 401301.
2. 200008011.
3. Ninety millions one hundred and five thousand and three.
4. 123455616.
5. £26825 7*s.*
6. 18586207.
7. £2832 17*s.* $7\frac{1}{4}$*d.*
8. 302290384623.
9. 159019937720.
10. £30018 4*s.* $9\frac{1}{4}$*d.*
11. £136853 7*s.* $1\frac{1}{2}$*d.*
12. £2577077 15*s.* 10*d.*
13. $114749405157\frac{1}{5}$.
14. $2045329251\frac{339}{341}$.
15. $21264174\frac{4523}{5037}$.
16. £223506 6*s.* $10\frac{1}{2}$*d.*
17. £23005 2*s.* 1*d.*

LIV.

1. **64 tons 6 cwts. 3 qrs. 14 lbs. 10 oz.**
2. **1*s*. 6*d*.**
3. **£1232 13*s*. $6\frac{1}{2}$*d*.**
4. **48516 inches.**
5. **4 miles $1378\frac{2}{3}$ yds.**
6. **£26 5*s*. $11\frac{1}{4}$*d*.**
7. **2520 half-crowns.**
8. **£500.**
9. **£39 14*s*. $9\frac{1}{2}$*d*.**

LV.

1. **125 lbs. 3 oz. 5 dwts. 12 grs.**
2. **48 lbs.**
3. **£953 14*s*. 8*d*.**
4. **£21 15*s*. 9*d*.**
5. **$15\frac{3}{7}$.**
6. **$4\frac{17}{18}$.**
7. **39.**
8. **10.**
9. **12988·76693.**
10. **446·352.**
11. **11961·232.**
12. **·0406.**
13. **£2 12*s*. 6*d*.**
14. **92 mls. 4 fur. 18 pls. 4 yds. 2 ft. 10 in.**
15. **$9\frac{1}{2}$ miles.**
16. **£20 13*s*.**
17. **£3098 6*s*. 2*d*.**
18. **$15\frac{1}{12}$.**
19. **$3\frac{3}{4}$.**
20. **35.**
21. **$\frac{7}{44}$.**
22. **51206·082.**
23. **55·0124.**
24. **318·906015.**
25. **245·538461.**
26. **8·625.**
27. **$221339\frac{1}{4}$ square feet.**
28. **$8\frac{7}{8}$ guineas (£9 6*s*. $4\frac{1}{2}$*d*.)**
29. **£113 17*s*. $3\frac{17}{18}$*d***
30. **16 years 8 months.**
31. **$9\frac{11}{12}$.**
32. **$3\frac{17}{44}$.**
33. **$37\frac{1}{2}$.**
34. **$2\frac{1}{2}$.**
35. **2000·83672.**
36. **865·49885.**
37. **6861·90045.**
38. **·00071...**
39. **·091954...**
40.

LVI.

1. **22188 grs.**
2. **$78\frac{1}{5}$ acres.**
3. **£3 10*s*. $8\frac{3}{4}$*d*.**
4. **£14 2*s*. 1·305*d*.**
5. **$2\frac{43}{80}$.**
6. **$\frac{47}{60}$.**
7. **$\frac{70}{221}$.**
8. **$1\frac{43}{280}$.**
9. **843·8985.**
10. **·3864.**
11. **112·9115.**
12. **·308...**
13. **£2 1*s*. 9*d*.**
14. **4 wks. 4 days 1 hr. 50 m. 43 sec.**

15. 4 weeks 2 days.
16.* £7 12*s*.
17. £3408 13*s*. 9*d*.
18. $12\frac{7}{48}$.
19. $2\frac{37}{48}$.
20. 32.
21. $\frac{25}{51}$.
22. 1002·46165.
23. ·907905.
24. 31·40899616.
25. 9640.
26. 1·71875.

LVII.

1. 8*s*. $4\frac{7100}{14329}$*d*.
2. $4941\frac{29}{38}$ cub. feet.
3. £1576 8*s*. 4·3212890625*d*.
4. £5 2*s*. $11\frac{135}{259}$*d*.
5. Tea 5*s*. and sugar 4*d*.
6. 736 men.
7. $6\frac{2}{3}$ years.
8. $94\frac{5}{8}$.
9. £715.

LVIII.

1. £3 3*s*. $10{\cdot}27\dot{6}\dot{3}$*d*.
2. £529 0*s*. $7\frac{1}{2}$*d*.
3. £885 12*s*. 7·93125*d*.
4. £858 18*s*. 2·7721875*d*.
5. $16\frac{33}{56} : 129\frac{19}{90}$.
6. 469·5102 : 444·144.
7. $\cdot 4017\dot{8}5714\dot{2}$.
8. 4*s*. 0·18996*d*.
9. 838·42.
10. 25 poles 2 yards $6\frac{4}{7}$ inches.
11. £14 3*s*. 4*d*.
12. 2*s*. 1*d*.
13. ·011 : ·0011 : 110.
14. 18 lbs. 8 oz.
15. $43\frac{3}{4}$.
16. 4*s*.
17. $\frac{13}{4000}$.
18. ·365625£.
19. $\frac{11}{12}$ of an inch.
20. Increases it 25 per cent.
21. $2\frac{22}{31}$*d*. : $2\frac{1}{31}$*d*. : and $1\frac{10}{31}$*d*.
22. 55 ft. 8″ 5‴ 2⁗.
23. 55 square feet $8\frac{31}{72}$ inches.
24. £169 6*s*. $4\frac{3}{4}$*d*.
25. £10330 18*s*. $2\frac{3}{4}$*d*.
26. £2700.
27. £57 12*s*.
28. £36.
29. £919 12*s*. 8·110521*d*.
30. $3\frac{6523}{7866}$.
31. 12*s*. $9\frac{9}{10}$*d* : ·64125£.
32. $\frac{33}{3125} : \frac{349}{2200}$.
33. 861.
34. £99 1*s*. $11\frac{4}{73}$*d*.
35. (1) £1431 3*s*. $2\frac{407}{504}$*d*.
    (2) £1415 1*s*. $6\frac{23}{56}$*d*.
    (3) £16 1*s*. $8\frac{25}{63}$*d*.
36. £128 1*s*. 0·860025*d*.
37. 11268 men.
38. 11*s*. $7\frac{23}{43}$*d*.
39. $\cdot 25658143\dot{9}$ — $\cdot 256581\dot{4}3\dot{9}$.
40. 25·828125.
41. Gain £5 13*s*. $9\frac{35}{41}$*d*.
42. Decrease ·03283....
43. $13\frac{1}{3}$ per cent.
44. $82\frac{1}{2}$.
45. 16*s*. 3*d*.
46. 11·18 per cent. nearly.

---

* The number of working days in a week is taken.

47. 7 minutes 3·36 seconds.
48. 10 gulden.
49. $3\frac{11}{90}$.
50. $\frac{1}{24}$.
51. 729 : $5\frac{2}{3}$.
52. See **174.**
53. 5·7948166 : 623·4478.
54. £297 15*s*. 3*d*.
55. £3 5*s*. $9\frac{47}{77}$*d*. : 1*s*. $9\frac{13}{231}$*d*.*
56. £16206 15*s*.
57. 9977664488.
58. 10915745·219638...
59. $\frac{1017}{1955}$.
60. $\cdot0\dot{6}$.
61. £517 17*s*. $1\frac{5}{7}$*d*.
62. £515 12*s*. $4\frac{1}{4}$*d*.
63. 1 pole 1 ft. $3\frac{1}{3}$ inches.
64. £24 2*s*. $3\frac{11}{47}$*d*.
65. 982.
66. 6*s*. $9\frac{21}{58}$*d*.
67. $5\frac{3}{4}$ per cent. gain.
68. 400.
69. ·104668... : ·11577...
70. £3 4*s*. $9\frac{81}{247}$*d*. : £5 5*s*. $9\frac{207}{397}$*d*.
71. 10 hrs. 30 mins.
72. Decrease 3·8374713.
73. 4 per cent.
74. 19 hrs. 20 mins.
75. 5 per cent gain.
76. $15\frac{15}{29}$ per cent.
77. 4·395...
78. $2\frac{1}{2}$.
79. $82\frac{1}{7}$.
80. 25 cubic ft. $866\frac{23}{48}$ inches.
81. 914 yds. 1 ft. 7 in.
82. $2\frac{7}{36}$.
83. $1\frac{1}{2}$ hour.
84. 10 hrs. 48 mins.
85. 1·25 lbs.
86. 9009.
87. 19 feet $3'\ 7''\ 3'''\ 2''''\ 10'''''$.
88. 19 cubic feet $519\frac{17}{72}$ inches.
89. (1) £44 2*s*. $4\frac{4}{17}$*d*. :
(2) £22 1*s*. $2\frac{2}{17}$*d*. : and
(3) £7 7*s*. $0\frac{12}{17}$*d*.
90. If its denominator was 2 or 5, or made up of twos or fives, see **101.**
91. See **100**.
92. Less than the units in denominator. See **102.**
93. (1) £62 4*s*. 7*d*. :
(2) £46 13*s*. $5\frac{1}{4}$*d*.
(3) £37 6*s*. 9*d*. :
(4) £31 2*s*. $3\frac{1}{2}$*d*.
94. $28\frac{925}{1406}$.
95. 1 year $153\frac{139}{147}$ days.
96. 2*s*. 4*d*.
97. $18\frac{2}{3}$ lbs.
98. $23464\cdot50\dot{3}$.
99. 18.
100. 17*s*. $2\frac{1}{4}$*d*.
101. $8\cdot73\dot{8}$*d*.
102. Increased $63\frac{1}{3}$ per cent.
103. £3 2*s*. $10\frac{698}{763}$*d*. :
£4 16*s*. $9\frac{9}{31}$*d*.
104. 50 per cent.
105. £30 1*s*. $6\frac{3}{4}$*d*. per cent.
106. £222896 14*s*. $5\frac{1}{4}$*d*.
107. £132787 18*s*. $9\frac{1}{2}$*d*.
108. $7\frac{1}{8}$.
109. £533 3*s*. 9*d*.
110. $1\frac{1}{7}$.
111. £1 1*s*. $6\frac{1}{16}$*d*. and £7 16*s*. $2\frac{286}{461}$*d*.
112. 1.

---

* In discounter's favour.

113. 14*s*. 6*d*.
114. £25 19*s*. $6\frac{153}{1168}$*d*.
115. £1129 3*s*. 4*d*.
116. 3 per cent.
117. $4\frac{1}{4}$ per cent.
118. $\cdot 13\dot{0}\dot{9}$.
119. 4 years 6 months.
120. 105.
121. $481\frac{43}{95}$ acres.
122. £4 10*s*.
123. £8508 10*s*. $10\frac{1}{2}$*d*.
124. ·2646£.
125. £181 1*s*. $5\frac{601}{2407}$*d*.
126. $\frac{54653}{90900}$.
127. $\frac{1}{42}$.
128. $2\frac{57}{176}$.
129. 3*s*. 6*d*.
130. ·007875.
131. $6\frac{3}{4}$ days.
132. £9 10*s*. $2\frac{2}{3}$*d*. per cent. gain.
133. 10 feet $2'\ 10''\ 7'''\ 3''''\ 9'''''$.
134. 10 ft. $415\frac{5}{192}$ inches.
135. 3 furlongs $27\frac{23}{27}$ pls.
136. £274 13*s*. 9*d*.
137. £7878 11*s*. 2*d*.
138. £137 1*s*. $4\frac{1}{2}$*d*.
139. $\frac{17}{65}$.
140. 21 days.
141. Nitre 30 cwts. 1 qr. $16\frac{4}{5}$ lbs.
Sulphur 3 cwts. 3 qrs. $5\frac{3}{5}$ lbs.
Charcoal 5 cwts. 3 qrs. $5\frac{3}{5}$ lbs.
142. £10020.
143. $37\cdot 58\dot{3}$.
144. £1305 18*s*. $0\frac{1}{4}$*d*.
145. 37 lbs.
146. Train goes $2\frac{1}{11}$ times as fast (23 : 11).
147. £248 5*s*. 4*d*.
148. 1 lb. 2 oz. 11 dwts. 16 grs.
149. $\frac{4}{15} : \frac{8243}{9900}$.
150. $685\frac{5}{7}$.
151. £65 12*s*. 6*d*.
152. £2700.
153. £6 9*s*. 10*d*.
154. £4173 16*s*. 3·624525*d*.
155. $6\frac{3}{8}$.
156. £15 0*s*. $11\frac{1}{2}$*d*. each.
157. £1000.
158. £14 8*s*.
159. $2\frac{3}{17}$.
160. ·8014...
161. $-1\cdot 9822713$.
162. 400000.
163. ·46553£.
164. 12800.
165. £7144 7*s*. 6*d*.
166. $\frac{67}{495} : \frac{17}{1375}$.
167. 74·782.
168. 98·581875 lbs.
169. $\frac{2630921}{6666660}$.
170. 10710·72246.
171. $\frac{13}{90} : \frac{8546}{24975}$.
172. £36 13*s*. 1·420905*d*.
173. Increased £99 5*s*. $6\frac{213}{262}$*d*. :
Gain per cent £44 2*s*. $5\frac{91}{131}$*d*.
174. £62 10*s*.
175. £350 5*s*.
176. $25\frac{85}{98}$.
177. 5*s*. 3*d*.
178. 30 oz.
179. $106\frac{2}{3}$.
180. 17*s*. 6*d*.
181. £17 18*s*. 11*d*.
182. £5 2*s*. 5*d*.
183. $\frac{841}{6900}$.
184. £20 12*s*. 1·9296*d*.
185. $\frac{8}{9}$.
186. ·06.
187. £1 5*s*. $2\frac{11}{14}$*d*.
188. 10100 shares.

189. $2\frac{2}{35}$ hrs.
190. $\frac{349}{986}$.
191. £1 7*s.*
192. £2 4*s.* $7\frac{5}{28}$*d.*
193. 4 per cent.
194. 300.
195. ·012$\dot{5}$30$\dot{8}$.
196. $\frac{6997}{18500}$.
197. 86.
198. 3620.
199. 837.
200. ·5.
201. 742.
202. $287466\frac{2}{3}$ solid yards.
203. 34 miles.
204. £565 13*s.* $7\frac{2908}{3125}$*d.*
205. 22 and 12.
206. 30 minutes.
207. $4\frac{1}{2}$ miles per hour.
208. 24 hrs. 3 mins. 55·71 ...secs.
209. $85\frac{5}{27}$.
210. £18630.
211. 9 hrs. 40 mins.
212. First $2\frac{1}{2}$, second $1\frac{1}{2}$ yrs.
213. 6*s.* $8\frac{1}{2}$*d.* nearly.
214. £62 0*s.* $4\frac{7}{8}$*d.*
215. £5040.
216. 36*s.* $3\frac{15}{19}$*d.*
217. £12 18*s.* $0\frac{72}{83}$*d.*
218. 40 per cent.
219. 27 inches.
220. $82\frac{2}{3}$ yards.
221. $2297\frac{101}{417}$ cubic ft.
222. 44·2718...
223. 165·6502...

LIX.

1. 26·1509125.
2. 18*s.* 9*d.* per gallon.
3. $22\frac{1}{8}$ years.
4. $5\frac{85}{123}$ (·$\dot{6}$910$\dot{5}$) gain.
5. 18495000.
6. £52 10*s.* increase.
7. $88\frac{6}{13}$.

LX.

1. $128\frac{10}{13}$.
2. Decreased $18\frac{74}{377}$ per cent.
3. £$7092\frac{28}{141}$ and £8 10*s.* $2\frac{26}{47}$*d.*
4. $82\frac{1}{7}$.
5. See the Question.
6. 10 gulden.
7. £1 19*s.* $4\frac{56}{127}$*d.*
8. £2 3*s.* $2\frac{52774}{85007}$*d.* gain.

LXI.

1. Train $1\frac{7}{8}$ times as fast (8 to 15).
2. $50\frac{3}{19}$.
3. 1·794...
4. 12 ft. 9′ 8′′′′ 11′′′′′.
5. 12 c. ft. $1296\frac{107}{144}$ (·7430$\dot{5}$) in.
6. 28·$\dot{6}\dot{3}$.
7. 26·543541$\dot{6}$.
8. Increased £29 16*s.* $7\frac{63}{263}$*d.*
9. 50 per cent.
10. £2 15*s.* 3*d.*
11. $66\frac{2}{3}$ men.
12. $17\frac{1}{9}$ days.
13. 3 mins. 46·8 secs.
14. 55 mins. past 10 o'clock.
15. $\frac{1}{7}$ of the work.

LXII.

1. 27720.
2. $\frac{4336}{58881}$.
3. 666 : ·00067 : ·6324...
4. 62·45614... : ·707106...
5. £792 15*s.*
6. The three and a half per cents.
7. He sees 30 yds. and travels twice as fast.
8. 4 dwts. $17\frac{6411}{25543}$ grs.
9. ·$\dot{1}$.
10. $65\frac{5}{27}$ yds.
11. $10\frac{1}{2}$*d.* per lb.
12. ·28 : 18000.
13. ·$\dot{0}4938271\dot{6}$: ·$8\dot{3}$.
14. £50000 : £7000.
15. 10 per cent.
16. $6\frac{4}{81}$ days.
17. $\bar{2}$·158362.
18. 0.
19. A, £128 3*s.* : B, £598 0*s.* 8*d.* : C, £170 17*s.* 4*d.* = $\frac{4}{21}$.
20. $7\frac{1}{2}$ yrs.
21. Inversely proportional to the price per gallon.
22. 176 tons 18 cwts. 2 qrs. $15\frac{3}{11}$ lbs.
23. £2124 15*s.* 9*d.*
24. 5 dwts. 10 grs.
25. $4\frac{23}{28}$.
26. 7*s.* $4\frac{1}{2}$*d.*
27. 736.
28. 64 days.
29. 8 horses.
30. 2·828 : ·014 : ·039592.
31. 4 lbs. 8 oz. 12 dwts. $19\frac{901}{1177}$ grs.
32. £1750.
33. ·$488541\dot{6}$.
34. $1\frac{1}{2}$: $\frac{2}{5}$: $2\frac{11}{20}$.
35. $\frac{7}{25}$.
36. £33 1*s.* $2\frac{2}{5}$*d.*
37. 1250 : ·125.
38. $7\frac{1}{4}$.
39. 2·1544345...
40. (1) $\frac{3}{100}$: (2) $\frac{231}{8000}$:
41. £325 10*s.*
42. $\sqrt{10}$ (3·162277...)
43. £1362 1*s.* 8*d.*
44. 14 yds. 1 ft.
45. £56 8*s.* 4*d.*
46. £1499 15*s.* 7·2*d.*
47. 15 days.
48. Income £900, and rate 5*d.*
49. 77 to 14.
50. £3 10*s.*
51. £900.
52. 237500 francs.

LONDON
PRINTED BY SPOTTISWOODE AND CO.
NEW-STREET SQUARE

GRADUATED SERIES OF ENGLISH READING-BOOKS.

New Editions, corrected, of the *Complete Series*, in 5 vols. fcp. 8vo. price 10*s*. cloth, each of which Volumes may be had separately,

THE GRADUATED SERIES
OF
# FIVE READING-LESSON BOOKS,
WITH EXPLANATORY NOTES;

Adapted, as a Progressive Course of Reading, for all Classes of English Schools and Families.

BY J. S. LAURIE,
Editor of the *Shilling Entertaining Library, &c.*

	s.	d.
FIRST BOOK, 192 Pages, *Seventh Edition*	1	0
SECOND BOOK, 256 Pages, *Sixth Edition*	1	6
THIRD BOOK, 312 Pages, *Sixth Edition*	2	0
FOURTH BOOK, 440 Pages, *Sixth Edition*	2	6
FIFTH BOOK, 496 Pages, *Second Edition*	3	0

THIS is an entirely new series of Reading-Books, carefully adapted throughout to the requirements of modern education. The Five Books are arranged each in corresponding sections, on a serial and uniform scheme of progressive, yet constantly varied, selections. BOOK I. consists of rhymes and fireside stories, fables and parables, and short simple tales, all within the comprehension of children who have mastered the first steps in reading. BOOK II. contains miscellanies, tales of adventure, imaginative and real, anecdotes in natural history, and ballad poetry—all preliminary to the Third Book. BOOK III. comprises literary selections in prose and verse, descriptive travel, natural history (with reference to the previous section), and narratives of English history. BOOK IV. to which the Third Book is introductory, is a further extension of the same general plan, with the addition of a division on the more popular branches of Natural Science and Physics, sequentially arranged. BOOK V. which completes the course, forms a further advance, and a completion of the general plan, and aims at answering the practical purposes of a Class-book of later English literature.

'MR. LAURIE's excellent *Graduated Series of Reading-Lesson Books* has met with great and well-deserved success.' ATHENÆUM.

By the same Author,

FIRST STEPS to READING: being an Introduction to the Graduated Series of English Reading-Books. Fcp. 8vo. PART I. price 3*d*. PART II. price 6*d*. sewed; or complete, price 10*d*. cloth. Or the whole consp[illegible] for Class Teaching on a Set of Broa[illegible]ce 7*s*. the Set of BROADSIDES mounte[illegible] with convenient IRON FRAME; the [illegible]s. 6*d*.

London: LONGMAN, [illegible] Row.